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SS 2019

Exercise Sheet 9 "Nonlinear Partial Differential Equations"
(parabolic PDEs)

Exercise 1. Let $\Omega = \mathbb{R}^3$ and $T > 0$. Consider the NAVIER-STOKES-equation

$$u_t + (u \cdot \nabla)u + \nabla p = \Delta u \ \text{ in } \ \Omega \times (0, T]\,. \tag{1}$$

Show that the NAVIER-STOKES equation supports the following symmetry groups:

(i) Galilean invariance: If (u, p) is a solution of (1) and $c \in \mathbb{R}^3$ is a constant vector, then

$$u_c(x,t) := u(x - ct, t) + c\,, \quad p_c(x,t) := p(x - ct, t)\,,$$

is a solution of (1).

(ii) Rotation symmetry: If (u, p) is a solution of (1) and $Q \in \mathbb{R}^{3\times 3}$ is a rotation matrix, i.e. $Q^T = Q^{-1}$, then

$$u_Q(x,t) := Q^T u(Qx, t)\,, \quad p_Q(x,t) := p(Qx, t)\,,$$

is a solution of (1).

(iii) Scale invariance: If (u, p) is a solution of (1) and $\tau \in \mathbb{R}_+$, then

$$u_\tau(x,t) := \tfrac{1}{\sqrt{\tau}} u\big(\tfrac{x}{\sqrt{\tau}}, \tfrac{t}{\tau}\big)\,, \quad p_\tau(x,t) := \tfrac{1}{\tau} p\big(\tfrac{x}{\sqrt{\tau}}, \tfrac{t}{\tau}\big)\,,$$

is a solution of (1).

Solutions will be discussed on Thursday 23th of May 2019.