

# PRAKTISCHE MATHEMATIK 2

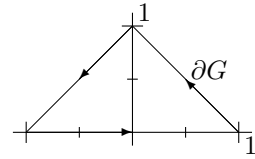
1. Test am 19. März 2007

## Gruppe A

- Berechnen Sie das Integral

$$\int_{\partial G} \left( \frac{y^2 + \tan(x-y)}{x^2 - \tan(x-y)} \right) \cdot dr$$

unter Zuhilfenahme eines Ihnen geeignet erscheinenden Integralsatzes.

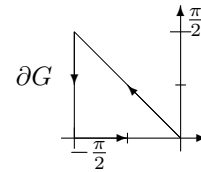


$$\begin{aligned} \int_{\partial G} \left( \frac{y^2 + \tan(x-y)}{x^2 - \tan(x-y)} \right) \cdot dr &= \int_0^1 dy \int_{y-1}^{1-y} dx \left( \frac{\partial}{\partial x} (x^2 - \tan(x-y)) - \frac{\partial}{\partial y} (y^2 + \tan(x-y)) \right) \\ &= \int_0^1 \int_{y-1}^{1-y} (2x - (1 + \tan^2(x-y)) - (2y + (1 + \tan^2(x-y)) \cdot (-1))) \, dx dy \\ &= \int_0^1 \int_{y-1}^{1-y} (2x - 2y) \, dx dy = \int_0^1 ((1-y)^2 - (y-1)^2 - 2y(1-y) + 2y(y-1)) \, dy \\ &= \int_0^1 (4y^2 - 4y) \, dy = 4 \left( \frac{y^3}{3} - \frac{y^2}{2} \right)_0^1 = 4 \left( \frac{1}{3} - \frac{1}{2} \right) = -\frac{2}{3}. \end{aligned}$$

## Gruppe B

- Verifizieren Sie den Satz von Green anhand des Integrals

$$\int_{\partial G} \begin{pmatrix} \cos x - \sin y \\ \sin x + \cos y \end{pmatrix} \cdot dr.$$



$$\begin{aligned} \int_G \begin{pmatrix} \cos x - \sin y \\ \sin x + \cos y \end{pmatrix} \cdot dr &= \int_0^{\pi/2} \begin{pmatrix} \cos(-t) - \sin t \\ \sin(-t) + \cos t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} dt + \int_{\pi/2}^0 \begin{pmatrix} \cos(-\frac{\pi}{2}) - \sin t \\ \sin(-\frac{\pi}{2}) + \cos t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} dt \\ &\quad + \int_{-\pi/2}^0 \begin{pmatrix} \cos t - \sin 0 \\ \sin t + \cos 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} dt \\ &= \int_0^{\pi/2} (-\cos(-t) + \sin t + \sin(-t) + \cos t) dt + \int_{\pi/2}^0 ((-1) + \cos t) dt + \int_{-\pi/2}^0 \cos t dt \\ &= (-t + \sin t) \Big|_{\pi/2}^0 + \sin t \Big|_{-\pi/2}^0 = \left( \frac{\pi}{2} - 1 \right) + (-(-1)) = \frac{\pi}{2}. \\ \int_G \left( \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right) \cdot dS &= \int_{-\pi/2}^0 dx \int_0^{-x} dy (\cos x + \cos y) = \int_{-\pi/2}^0 (y \cos x \Big|_0^{-x} + \sin y \Big|_0^{-x}) dx = \int_{-\pi/2}^0 (-x \cos x - \sin x) dx \\ &= (-\cos x - x \sin x + \cos x) \Big|_{-\pi/2}^0 \\ &= -\cos 0 - 0 \cdot \sin 0 + \cos 0 - \left( -\cos\left(-\frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right) \sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) \right) \\ &= -\frac{\pi}{2} \sin\left(-\frac{\pi}{2}\right) = \frac{\pi}{2}. \end{aligned}$$

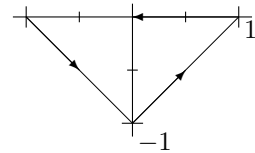
## Gruppe C

- Berechnen Sie das Integral

$$\int_{\partial K} \left( \frac{y^2 - \sin x}{x^2 + \cos y} \right) \cdot dr$$

unter Zuhilfenahme eines Ihnen geeignet erscheinenden Integralsatzes.

$\partial G$

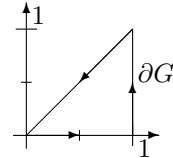


$$\begin{aligned} \int_{\partial G} \left( \frac{y^2 - \sin x}{x^2 + \cos y} \right) \cdot dr &= \int_{-1}^0 dy \int_{-y-1}^{y+1} dx \left( \frac{\partial(x^2 + \cos y)}{\partial x} - \frac{\partial(y^2 - \sin x)}{\partial y} \right) \\ &= \int_{-1}^0 \int_{-y-1}^{y+1} (2x - 2y) dx dy = \int_{-1}^0 (x^2 - 2xy) \Big|_{-y-1}^{y+1} dy \\ &= \int_{-1}^0 ((y+1)^2 - 2y(y+1) - (y+1)^2 + 2y(-y-1)) dy \\ &= \int_{-1}^0 (-2y(y+1) + 2y(-y-1)) dy = -4 \int_{-1}^0 (y^2 + y) dy \\ &= -4 \left( \frac{y^3}{3} + \frac{y^2}{2} \right) \Big|_{-1}^0 = \frac{2}{3}. \end{aligned}$$

## Gruppe D

- Verifizieren Sie den Satz von Green anhand des Integrals

$$\int_{\partial G} \left( \frac{x^2 y^2}{x^2 + y^2} \right) \cdot dr.$$



$$\begin{aligned} \int_{\partial G} \left( \frac{x^2 y^2}{x^2 + y^2} \right) \cdot dr &= \int_0^1 \underbrace{\begin{pmatrix} 0 \\ t^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{=0} dt + \int_0^1 \underbrace{\begin{pmatrix} t^2 \\ 1+t^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{=1+t^2} dt + \int_1^0 \underbrace{\begin{pmatrix} t^4 \\ 2t^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{=t^4+2t^2} dt \\ &= \int_0^1 (0 + 1 + t^2 - t^4 - 2t^2) dt = t - \frac{t^3}{3} - \frac{t^5}{5} \Big|_0^1 = \frac{7}{15} \\ \int_G \left( \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right) \cdot dS &= \int_0^1 dx \int_0^x dy (2x - 2x^2 y) = \int_0^1 (2xy \Big|_0^x - x^2 y^2 \Big|_0^x) dx \\ &= \int_0^1 (2x^2 - x^4) dx = 2 \frac{x^3}{3} - \frac{x^5}{5} \Big|_0^1 = \frac{7}{15}. \end{aligned}$$