

PRAKTISCHE MATHEMATIK 2

3. Test am 28. April 2008

Gruppe A,C

- Finden Sie jene Funktion $y(x)$ mit $y(x_1) = d_1$, $y(x_2) = d_2$ für die

$$\int_{x_1}^{x_2} (a^2(y')^2 - b^2 y^2) dx$$

extremal wird.

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$$\frac{d}{dx} \frac{\partial}{\partial y'} = \frac{\partial}{\partial y} \Rightarrow 2a^2 y'' = -2b^2 y \Rightarrow y'' + \frac{b^2}{a^2} y = 0$$

$$\Rightarrow y = c_1 \sin\left(\frac{b}{a}x\right) + c_2 \cos\left(\frac{b}{a}x\right)$$

$$c_1 \sin\left(\frac{b}{a}x_1\right) + c_2 \cos\left(\frac{b}{a}x_1\right) = d_1$$

$$c_1 \sin\left(\frac{b}{a}x_2\right) + c_2 \cos\left(\frac{b}{a}x_2\right) = d_2$$

$$\Rightarrow c_1 = \begin{cases} c_1 = \frac{1}{\sin\left(\frac{b}{a}x_1\right)} (d_1 - c_2 \cos\left(\frac{b}{a}x_1\right)) & x_1 \neq k\pi, k \in \mathbb{Z} \\ c_2 = (-1)^k d_1 & x_1 = k\pi \end{cases}$$

$$\Rightarrow \frac{\sin\left(\frac{b}{a}x_2\right)}{\sin\left(\frac{b}{a}x_1\right)} \left(d_1 - c_2 \cos\left(\frac{b}{a}x_1\right)\right) + c_2 \cos\left(\frac{b}{a}x_2\right) = d_2, \quad x_1 \neq k\pi$$

$$\Rightarrow c_2 = \frac{d_2 \sin\left(\frac{b}{a}x_1\right) - d_1 \sin\left(\frac{b}{a}x_2\right)}{\sin\left(\frac{b}{a}x_1\right) \cos\left(\frac{b}{a}x_2\right) - \cos\left(\frac{b}{a}x_1\right) \sin\left(\frac{b}{a}x_2\right)}, \quad x_1 \neq k\pi$$

$$\Rightarrow c_1 = \frac{d_1}{\sin\left(\frac{b}{a}x_1\right)} - \frac{\cos\left(\frac{b}{a}x_1\right) \left(d_2 - d_1 \frac{\sin\left(\frac{b}{a}x_2\right)}{\sin\left(\frac{b}{a}x_1\right)}\right)}{\sin\left(\frac{b}{a}x_1\right) \cos\left(\frac{b}{a}x_2\right) - \cos\left(\frac{b}{a}x_1\right) \sin\left(\frac{b}{a}x_2\right)}, \quad x_1 \neq k\pi$$

$$\Rightarrow c_1 = \frac{d_2 - (-1)^k \cos\left(\frac{b}{a}x_2\right)}{\sin\left(\frac{b}{a}x_2\right)}, \quad x_1 = k\pi$$

Gruppe B,D

- Finden Sie jene Funktion $y(x)$ mit $y(x_1) = d_1$, $y(x_2) = d_2$ für die

$$\int_{x_1}^{x_2} (a^2(y')^2 + b^2 y^2) dx$$

extremal wird.

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$$\frac{d}{dx} \frac{\partial}{\partial y'} = \frac{\partial}{\partial y} \quad \Rightarrow \quad 2a^2 y'' = 2b^2 y \quad \Rightarrow \quad y'' - \frac{b^2}{a^2} y = 0$$

$$\Rightarrow \quad y = c_1 e^{\frac{b}{a} x} + c_2 e^{-\frac{b}{a} x}$$

$$c_1 e^{\frac{b}{a} x_1} + c_2 e^{-\frac{b}{a} x_1} = d_1$$

$$c_1 e^{\frac{b}{a} x_2} + c_2 e^{-\frac{b}{a} x_2} = d_2$$

$$\Rightarrow \quad c_1 = d_1 e^{-\frac{b}{a} x_1} - c_2 e^{-2\frac{b}{a} x_1}$$

$$\Rightarrow \quad c_2 = \frac{d_2 e^{\frac{b}{a} x_1} - d_1 e^{\frac{b}{a} x_2}}{e^{-\frac{b}{a}(x_2-x_1)} - e^{\frac{b}{a}(x_2-x_1)}}$$

$$\Rightarrow \quad c_1 = d_1 e^{-\frac{b}{a} x_1} - e^{-2\frac{b}{a} x_1} \frac{d_2 e^{\frac{b}{a} x_1} - d_1 e^{\frac{b}{a} x_2}}{e^{-\frac{b}{a}(x_2-x_1)} - e^{\frac{b}{a}(x_2-x_1)}}$$