1.3 Exercises 3: Date 15.5.2015

1. Using the data file **cps08_500.xls** now test the wage equation for heteroscedasticity using the **White test**. Re-estimate the equation

$$AHE_i = \beta_0 + \beta_1 AGE_i + \beta_2 BA_i + \beta_3 FE_i + u_i \qquad i = 1, ..., 500$$

calculate the residuals and save their squares e_i^2. Calculate the squares and cross products of the explanatory variables. Now explain each series of the **squared** residuals by regressing on a constant, the original explanatory variables and their squares and cross products. E.g. for two explanatory variables x and y the test equation will be

$$e_i^2 = b_0 + b_1 x_i + b_2 y_i + b_3 x_i^2 + b_4 y_i^2 + b_5 x_i y_i + u_i.$$

Be careful to avoid multicollinearity! Under the null of homoscedasticity the number of observations (n) times the R^2 of the test equation is distributed χ_q^2 with q the number of regressors in the test equation minus one. Reject the null if nR^2 is larger than the critical level of the respective χ^2distribution.

2. Estimate the wage equation with AHE as dependent variable with a **heteroscedasticity consistent variance estimator** (i.e. use the options in the regression software as e.g. in GRETL). Which differences do you observe?

3. Data for a monopolist's total revenue (R), total cost (C), and output (q), for 48 consecutive months can be found in the file **firmdata.xls.** Suppose that the monopolist's economic models for total revenue and total costs are given, respectively, by

$$\begin{aligned} R_t &= \beta_1 q_t + \beta_2 q_t^2 \qquad t = 1, ..., 48 \\ C_t &= \alpha_1 + \alpha_2 q_t + \alpha_3 q_t^2 \qquad t = 1, ...48 \end{aligned} \tag{6}$$

   (a) Find the profit maximizing level of output as a function of the unknown parameters.

   (b) Use the least squares estimator to estimate these parameters. For what statistical model are these estimates appropriate?

   (c) What do the least squares estimates suggest is the profit-maximizing level of output?

4. Separately test the residuals of these equations (6) to see if their errors might be autocorrelated. Calculate the empirical **autocorrelation function** r_k of the residuals, i.e. calculate the empirical correlation of the residuals e_t with the residuals shifted by k periods e_{t-k} for $k = 1, 2, ..., 10$ (see lecture notes p.28). If the true $\rho_k = 0$ the estimated values should lie between $\pm\frac{2}{\sqrt{T}}$ with 95% probability. Plot k on the x-axis against r_k on the y-axis and draw the lines $\pm\frac{2}{\sqrt{T}}$. Is the assumption of uncorrelated errors for this model justified?

5. Now test the specific hypothesis of first order autocorrelation of the error term, i.e.

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{7}$$

with ε_t a pure random term with mean $E(\varepsilon_t) = 0$, constant variance $E(\varepsilon_t^2) = \sigma_\varepsilon^2$ for all t, and $Cov(e_t e_s) = 0$ for $s \neq t$.
Compute the Durbin-Watson statistic for the residuals of each of the equations (6).
Perform the DW-test by testing the following hypotheses:
a) $\rho > 0$, against the alternative $\rho \leq 0$.
b) $\rho < 0$, against the alternative $\rho \geq 0$.

6. Use the Breusch and Godfrey test to test the residuals of the revenue equation (6) for autocorrelation up to order 8 (see lecture notes p.27).

7. Find a solution to the inhomogeneous first order difference equation (7) under the condition that $|\rho| < 1$ with a given disturbance function ε_t.

8. Assuming that ε_t is a pure random term with mean $E(\varepsilon_t) = 0$, constant variance $E(\varepsilon_t^2) = \sigma_\varepsilon^2$ for all t and $Cov(e_t e_s) = 0$ for $s \neq t$, show that for equation (7) under the condition that $|\rho| < 1$, the variance of u_t is equal to

$$E(u_t^2) = \frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{8}$$

   Hint: Use the answer to question 7 in the derivation.

9. Show that for equation (7) under the condition that $|\rho| < 1$ and the assumptions of question 8, the covariance between u_t and u_{t-1} is equal to

$$E(u_t u_{t-1}) = \frac{\rho\sigma_\varepsilon^2}{1-\rho^2}. \tag{9}$$

10. What are the consequences of having autocorrelated or heteroskedastic residuals for the statistical properties of the ordinary least squares estimator?.