

5. Übung Höhere Wahrscheinlichkeitstheorie

1. Let X be a topological group. What can be said about

$$AB = \{xy : x \in A, y \in B\}$$

if A and B each are open, compact, or closed?

In the following problems, let μ be a left invariant Haar measure on the locally compact group X .

2. Prove that every open set has positive measure.
3. If $f \in C_c^+ X$, $f \neq 0$, then $\int f d\mu > 0$.
4. Show that $\mu_a(A) = \mu(Aa)$ is left invariant. Conclude that there is a function $\gamma(a)$ such that

$$\mu(Aa) = \gamma(a)\mu(A).$$

γ is called the *modular function* of X .

5. For integrable f ,

$$\int f \circ R_a d\mu = \gamma(a^{-1}) \int f d\mu.$$

6. $\gamma(xy) = \gamma(x)\gamma(y)$ and γ is continuous.
7. $\nu_1(A) = \mu(A^{-1})$ and

$$\nu_2(A) = \int_A \frac{1}{\gamma(x)} d\mu(x)$$

are right invariant.