6. Übung Höhere Wahrscheinlichkeitstheorie

1. Follow-up to last week: in a locally compact group the product KA with K compact, A closed is closed (consider an accumulation point x of KA, compact neighborhood V of x and $A' = A \cap K^{-1}V$).

2. Let X be a random variable with $\phi_X(t) = 1$ for some $t \neq 0$. Show that with probability one, X takes values in $\frac{(2\pi}{t)\mathbb{Z}}$.

3. Prove:if X and Y are independent and X and $X + Y$ have the same distribution, then $Y = 0$ with probability one.

4. As a process with independent increments and zero expectation, the Wiener process β is a martingale. Prove that $\beta(t)^2 - t$ and $\eta(t) = e^{a\beta(t) - a^2 t/2}$ are martingales, too.

5. Apply the optional stopping theorem to $\eta(t) = e^{a\beta(t) - a^2 t/2}$ and calculate $\mathbb{E}(e^{-z\tau})$ for the stopping time
$$\tau = \inf\{t : \beta(t) = c\}.$$

6. Apply the optional stopping theorem to $\eta(t) = e^{a\beta(t) - a^2 t/2}$ and calculate $\mathbb{E}(e^{-z\tau})$ for the stopping time
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7. Apply the optional stopping theorem to $\eta(t) = e^{a\beta(t) - a^2 t/2}$ and calculate $\mathbb{E}(e^{-z\tau})$ for the stopping time
$$\tau = \inf\{t : \beta(t) = c - dt\} \ (c, d > 0).$$