8. Übung Höhere Wahrscheinlichkeitstheorie

1. Which of the well-known Banach spaces (e.g. $L_p, \ell_p, \mathbf{C}([0,1]), \mathfrak{M}([0,1], \mathfrak{B})$, ad libitum expandable, also as far as the underlying spaces are concerned).

2. Let μ be a regular finite measure on a Banach space. Prove that there is a separable closed subspace of full measure and for any $\epsilon > 0$ a finite-dimensional subspace E with $\mu(U(E,\epsilon)^C) < \epsilon$.

3. Let X be a metric space, $\tilde{X}$ its completion. Show that for a finite measure μ on $\mathfrak{B}(X)$,
$$\tilde{\mu}(A) = \mu(A \cap X)$$
defines a measure on $\mathfrak{B}(\tilde{X})$. If μ is regular, then so is $\tilde{\mu}$, and μ_n converges (weakly) to μ, iff $\tilde{\mu}_n$ converges to $\tilde{\mu}$.

4. The set of finite Radon measures is a closed subset (if signed measures are allowed, a closed subspace) of $(\mathfrak{M}, \|.\|_v)$.

5. In a metric space X, the point measures δ_x for $x \in X$ are Radon (no na), and $\delta_{x_\alpha} \Rightarrow \delta_x$ iff $x_\alpha \to x$.