

Höhere Wahrscheinlichkeitstheorie Markov Processes

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Exercise 1

Assume that S is a finite set, m a reference measure with support S , let Q be a Q -matrix and set $P_t := \exp(tQ)$, $\gamma_t := dP_t/dm$. Derive, discuss, and interpret *Kolmogorov forward equations* for

- (a) the distribution $\mu_t \sim X_t, t \geq 0$ (of a process corresponding to P_t), given $X_0 \sim \mu_0$.
- (b) $\gamma_t, t \geq 0$.

Exercise 2

Given transition matrices of a time-inhomogeneous Markov chain, i.e. matrices $P_{s,t}, 0 \leq s \leq t$ such that

- (a) $P_{s,t}P_{t,r} = P_{s,r}$,
- (b) $P_{s,s} = Id$,
- (c) $(s, t) \mapsto P_{s,t}$ is continuous,
- (d) $Q_s := \lim_{\Delta s \downarrow 0} 1/\Delta s [P_{s,s+\Delta s} - P_s]$ exists and is continuous in s ,

establish a Kolmogorov forward equation.