4. Tutorium - Resultate **15.04.2016**

4.1 Geladene Stäbe

a)

$E_y(x,y,0) = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{a-x}{y\sqrt{(x-a)^2+y^2}} + \frac{x}{y\sqrt{x^2+y^2}}\right]$.

$F_y(y=b) = \frac{\lambda_1\lambda_2}{4\pi\epsilon_0}\frac{2}{b}\left(\sqrt{a^2+b^2}-b\right)$.

$a \gg b$: $F_y \to \frac{q_1 q_2}{4\pi\epsilon_0}\frac{2}{ab}$.

$a \ll b$: $F_y \to \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{b^2}$

b) $\vec{F}_3(0,\frac{b}{2},c) = \frac{\lambda_1\lambda_3}{2\pi\epsilon_0}\begin{pmatrix}0\\0\\2c\end{pmatrix}\frac{1}{\frac{b^2}{4}+c^2}\left(\sqrt{a^2+\frac{b^2}{4}+c^2}-\sqrt{\frac{b^2}{4}+c^2}\right)$.

$a \ll \sqrt{(b/2)^2+c^2}$: $c = \pm\frac{b}{2\sqrt{2}}$.

Maximale Kraft: $F_{3,z}(0,\frac{b}{2},\pm\frac{b}{2\sqrt{2}}) = \frac{q_1 q_3}{\pi\epsilon_0}\vec{e}_z\frac{4}{3\sqrt{3}}\frac{1}{b^2}$.

$a \gg \sqrt{(b/2)^2+c^2}$: $c = \pm\frac{b}{2}$.

Maximalkraft: $F_{3,z}(0,\frac{b}{2},\pm\frac{b}{2}) = \frac{q_1 q_3}{\pi\epsilon_0}\vec{e}_z\frac{1}{ab}$.

4.2 Zylinder mit exzentrischer Längsbohrung

a) $V = \begin{cases}-\frac{\rho_0 r^2}{4\epsilon_0}+C_3 & r \le R_0\\ -\frac{\rho_0 R_0^2}{2\epsilon_0}\ln\frac{r}{R_0}-\frac{\rho_0 R_0^2}{4\epsilon_0}+C_3 & r > R_0\end{cases}$ $\quad\vec{E}(\vec{r}) = \vec{e}_r\begin{cases}\frac{\rho_0 r}{2\epsilon_0} & r \le R_0\\ \frac{\rho_0}{2\epsilon_0}\frac{R_0^2}{r} & r > R_0.\end{cases}$

b) $V(x,y,z) = \frac{\rho_0}{4\epsilon_0}\left(a^2-2ax\right)$.

$\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}) = \begin{pmatrix}\frac{a\rho_0}{2\epsilon_0}\\0\\0\end{pmatrix} = \frac{a\rho_0}{2\epsilon_0}\vec{e}_x = \frac{\vec{a}\rho_0}{2\epsilon_0}$.

4.3 Geladene Kugel

$E(r) = \frac{\rho_0}{4\epsilon_0}\begin{cases}\frac{r^2}{R} & r < R\\ \frac{R^3}{r^2} & r > R\end{cases} = \frac{Q}{4\pi\epsilon_0}\begin{cases}\frac{r^2}{R^4} & r < R\\ \frac{1}{r^2} & r > R\end{cases}$.

$Q = \pi R^3\rho_0$