# 2. Plenum, Lösung

EW, EV von $\hat{H}$

a) $\hat{H}^{\{S_z\}} = -g \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \Rightarrow \boxed{\begin{aligned} \lambda_\pm &= \pm g \\ |\pm\rangle &= \frac{1}{\sqrt{2}}\left(\pm i|1\rangle + |2\rangle\right) \end{aligned}}$

$\rightarrow$ Zeitentwicklung: $|\psi(t^*)\rangle = e^{-\frac{iHt^*}{\hbar}}|\psi(0)\rangle = e^{-\frac{iHt^*}{\hbar}}|2\rangle =$

$$= \frac{1}{\sqrt{2}} e^{-\frac{iHt^*}{\hbar}}\left(|+\rangle + |-\rangle\right) = \frac{1}{2}\Big[i\left(e^{-igt^*/\hbar} - e^{\frac{igt^*}{\hbar}}\right)|1\rangle + \left(e^{-\frac{igt^*}{\hbar}} + e^{\frac{igt^*}{\hbar}}\right)|2\rangle\Big] =$$

$$= \sin\left(\frac{gt^*}{\hbar}\right)|1\rangle + \cos\left(\frac{gt^*}{\hbar}\right)|2\rangle$$

b) $\langle S_z(t^*)\rangle = \langle\psi(t^*)|\hat{S}_z|\psi(t^*)\rangle = \frac{\hbar}{2}\Big[\langle\psi(t^*)|1\rangle\langle 1|\psi(t^*)\rangle - \langle\psi(t^*)|2\rangle\langle 2|\psi(t^*)\rangle\Big] =$

$$= \underline{\underline{\frac{\hbar}{2}\left[\sin^2\left(\frac{gt^*}{\hbar}\right) - \cos^2\left(\frac{gt^*}{\hbar}\right)\right]}}$$

Messwerte von $S_z$: $S_z^{\{S_z\}} = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow$ diagonal!

$\Rightarrow$ EW = Messwerte $= \underline{\underline{\pm\frac{\hbar}{2}}}$

Wahrscheinlichkeiten der Ergebnisse: $\begin{cases} P_{+\frac{\hbar}{2}} = |\langle 1|\psi(t^*)\rangle|^2 = \sin^2\left(\frac{gt^*}{\hbar}\right) \\ P_{-\frac{\hbar}{2}} = |\langle 2|\psi(t^*)\rangle|^2 = \cos^2\left(\frac{gt^*}{\hbar}\right) \end{cases}$

$\mathcal{E})$ $S_x^{\{S_z\}} = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ → nicht-diagonal in $S_z$-Basis!

$\lambda_{A,B} = \pm\frac{\hbar}{2}$ ⇒ mögliche Messwerte (Eigenwerte!) $\begin{cases} |A\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \\ |B\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \end{cases}$

-) falls $S_z = +\frac{\hbar}{2}$ bei $t = t^*$ → $|\psi\rangle = |1\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$

$$P_{S_x = \frac{1}{2}} = P_{S_x = -\frac{1}{2}} = \underline{\underline{\frac{1}{2}}}$$

-) falls $S_z = -\frac{\hbar}{2}$ bei $t = t^*$ → $|\psi\rangle = |2\rangle = \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle)$

$$P_{S_x = \frac{1}{2}} = P_{S_x = -\frac{1}{2}} = \underline{\underline{\frac{1}{2}}}$$

$$\Rightarrow P(S_z = +\tfrac{\hbar}{2}, S_x = -\tfrac{\hbar}{2}) = \sin^2\left(\frac{gt^*}{\hbar}\right)\frac{1}{2}$$

$$P(S_z = +\tfrac{\hbar}{2}, S_x = +\tfrac{\hbar}{2}) = \sin^2\left(\frac{gt^*}{\hbar}\right)\frac{1}{2} \qquad \sum P = 1 \checkmark$$

$$P(S_z = -\tfrac{\hbar}{2}, S_x = -\tfrac{\hbar}{2}) = \cos^2\left(\frac{gt^2}{\hbar}\right)\frac{1}{2}$$

$$P(S_z = -\tfrac{\hbar}{2}, S_x = +\tfrac{\hbar}{2}) = \cos^2\left(\frac{gt^*}{\hbar}\right)\frac{1}{2}$$