12. The interaction with the Z-boson can be expressed in the form

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W}\, J_\mu^{NC} Z^\mu\,, \tag{25}$$

where J_μ^{NC} is named the neutral current. The original relevant Lagrangian density of the GWS model is

$$\mathcal{L} = i\bar{L}\gamma_\mu D^\mu L + i\bar{R}\gamma_\mu D^\mu R\,, \tag{26}$$

where the letters L and R stand for the left-handed dublet ψ_L and the right-handed singlet ψ_R of one fermion generation, e.g. (ν_e, e_L^-) and e_R^-, and the operator D_μ acts on L and R as

$$D^\mu L = \left(\partial^\mu \mathbb{1}_{2x2} - ig\, T^a A^{a\mu} - ig'\, \frac{Y}{2}\mathbb{1}_{2x2} B^\mu\right) L\,; \quad D^\mu R = \left(\partial^\mu - ig'\, \frac{Y}{2} B^\mu\right) R\,. \tag{27}$$

Extract from the Lagrangian density Eq. (26) the part

$$g J_\mu^3 A^{3\mu} + \frac{1}{2} g' J_\mu^Y B^\mu \tag{28}$$

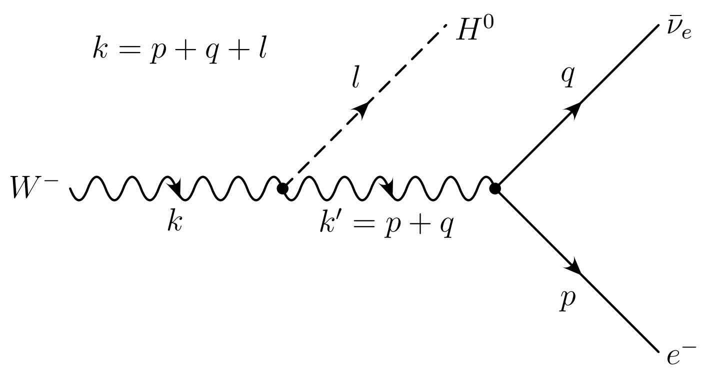
and perform the orthogonal transformation (22). Further use

$$J_\mu^{em} = J_\mu^3 + \frac{1}{2} J_\mu^Y \tag{29}$$

and show that

$$J_\mu^{NC} = J_\mu^3 - \sin^2\theta_W\, J_\mu^{em}\,. \tag{30}$$

13. Given is the decay of a real W boson, $W \to W^* + H^0 \to e\,\bar{\nu} + H^0$, where first a H^0 is radiated off.



The corresponding Lagrangians are

$$\mathcal{L}_{HWW} = g m_W\, H^0 W_\mu^+ W^{\mu -} \tag{31}$$

$$\mathcal{L}^{CC} = \frac{g}{\sqrt{2}} J_\mu^{CC} W^{\mu +} + h.c. \tag{32}$$

with $J_\mu^{CC} = \frac{1}{2}\,\bar{\nu}_e\gamma_\mu(1-\gamma_5)e$. Note that the W^+ field operator annihilates a positively charged W boson or creates a negatively charged one. Calculate the matrix element

$$\mathcal{M} = \frac{ig^2 m_W}{2\sqrt{2}}\, \frac{1}{(p+q)^2 - m_W^2} \bar{u}(p)\gamma_\mu(1-\gamma_5)v(q)\,\epsilon^\mu\,. \tag{33}$$

Furthermore, perform the summations over all spin states and the polarizations, with which you get

$$\overline{|\mathcal{M}|^2} = \frac{1}{3}\sum_{s,\lambda} |\mathcal{M}|^2 = \frac{g^4 m_W^2}{3\left[(p+q)^2 - m_W^2\right]^2} \left\{ (p.q) + \frac{2(p.k)(q.k)}{m_W^2} \right\} \tag{34}$$

For the calculation of the decay width use the convention

$$(k-p)^2 = t\,, \quad (k-q)^2 = u\,, \quad (k-l)^2 = s\,, \tag{35}$$

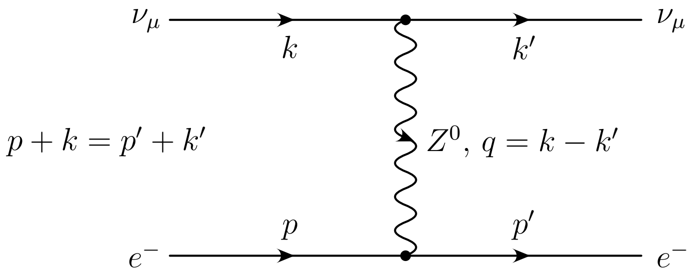
and the formula for the three-particle phase space,

$$\mathrm{dLips} = \frac{1}{128\pi^3 m_W^2} \int ds\, dt\,. \tag{36}$$

Consider the kinematic limits of the integration variables based on the results of example 15 (WS) and give the result for the differential width $\frac{\mathrm{d}^2\Gamma}{\mathrm{d}s\,\mathrm{d}t}$. Furthermore make the assumption $m_H \ll m_W$ (we know, experimentally $m_H > m_W$) and show that in this approximation the result for $\mathrm{d}\Gamma/\mathrm{d}s$ is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}s} = \frac{g^4}{768\pi^3 m_W} \left(-\frac{1}{12} + \frac{s}{12\, m_W^2} + \frac{s}{s - m_W^2} \right)\,. \tag{37}$$

14. We have the reaction $e^- + \nu_\mu \to e^- + \nu_\mu$.



Extract the necessary Feynman rules from the Lagrangian density

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} J_\mu^{NC} Z^\mu\,, \tag{38}$$

where

$$J_\mu^{NC} = \frac{1}{2}\left[g_L^f \bar{f}\gamma_\mu(1-\gamma_5)f + g_R^f \bar{f}\gamma_\mu(1+\gamma_5)f \right] \tag{39}$$

and

$$g_L^f = T_3(f_L) - Q(f_L)\sin^2\theta_W\,, \tag{40}$$

$$g_R^f = T_3(f_R) - Q(f_R)\sin^2\theta_W\,. \tag{41}$$