Exercises to Introduction to Models of Elementary Particle Physics II

141.B03, H. Eberl, SS 2021, v140721, draft

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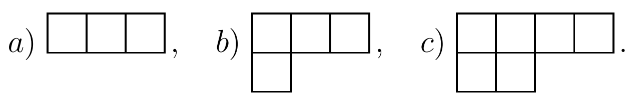
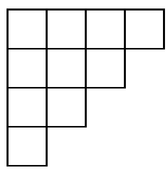
1. Given are the SU(3) Young tableaux



(1)

Calculate the dimension of these tableaux.
Draw the Young tableaux of the conjugate representation for a), b) and c) and check, that these tableaux have the same dimension as the ones given in (1).

2. Given is the SU(5) Young tableau



(2)

Calculate the dimension of this tableaux and of its complex conjugated representation. The results are the same. What does it mean?

3. Verify for the SU(3) representation

$$
\begin{aligned}
6 \otimes 3 &= 10 \oplus 8\,, && (3)\\
8 \otimes 8 &= 27 \oplus 10 \oplus 10^* \oplus 8 \oplus 8 \oplus 1\,, && (4)
\end{aligned}
$$

by using Young tableaux.

4. When multiplying basis vectors $\psi_\mu^{(\alpha)}$ in the representation space of the irreducible representations (α indicates the dimension, μ numbers the basis vectors), the obtained tensors are in general not basis tensors of the irreducible representation. However, the basis can be expressed as a linear combination of these product tensors. The coefficients occurring in the sum are then the Clebsch-Gordan coefficients.

Calculate the Clebsch-Gordan coefficients of the irreducible representation of $3 \otimes 2$ of SU(2) using the Young tableau formalism. First show that

$$3 \otimes 2 = 4 \oplus 2$$

and use as basis of 3

$$\begin{aligned} \psi_1^{(3)} &= u_1 u_1 \\ \psi_2^{(3)} &= (u_1 u_2 + u_2 u_1)/\sqrt{2} \\ \psi_3^{(3)} &= u_2 u_2, \end{aligned} \tag{5}$$

where $\psi_i^{(2)} = u_i,\ i = 1, 2$ is the basis in the representation space of the fundamental representation of SU(2). Express the irreducible representation of $3 \otimes 2$ in the basis $\psi_\mu^{(\alpha)} u_i$. The solutions are:

$$\begin{aligned} \psi_1^{(4)} &= \psi_1^{(3)} u_1 \\ \psi_2^{(4)} &= \sqrt{\frac{2}{3}} \psi_2^{(3)} u_1 + \sqrt{\frac{1}{3}} \psi_1^{(3)} u_2 \\ \psi_3^{(4)} &= \sqrt{\frac{2}{3}} \psi_2^{(3)} u_2 + \sqrt{\frac{1}{3}} \psi_3^{(3)} u_1 \\ \psi_4^{(4)} &= \psi_3^{(3)} u_2 \\ \psi_1^{(2)} &= -\sqrt{\frac{1}{3}} \psi_2^{(3)} u_1 + \sqrt{\frac{2}{3}} \psi_1^{(3)} u_2 \\ \psi_2^{(2)} &= \sqrt{\frac{1}{3}} \psi_2^{(3)} u_2 - \sqrt{\frac{2}{3}} \psi_3^{(3)} u_1. \end{aligned} \tag{6}$$

5. We assume a global SU(3) with basis vector (u_1, u_2, u_3). Calculate all irreducible representations of $3^* \otimes 3^* \otimes 3^*$. Create all the states of the individual multiplets using the Young tableaux formalism and derive formulas for calculating the individual states.

6. Consider QED as Abelian gauge theory. The photon field fulfils the transformation property
$$V'_\mu = V_\mu + \frac{1}{g} \partial_\mu \lambda(x). \tag{7}$$
Write down how a mass term for the photon field should look like and check whether it is gauge invariant then.

7. The kinetic term for the gauge field in QED is
$$\mathcal{L}_V = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{8}$$
Show its local gauge invariance.