12. The interaction with the Z-boson can be expressed in the form

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W}\, J_\mu^{NC} Z^\mu \,, \tag{25}$$

where J_μ^{NC} is named the neutral current. The original relevant Lagrangian density of the GWS model is

$$\mathcal{L} = i\bar{L}\gamma_\mu D^\mu L + i\bar{R}\gamma_\mu D^\mu R \,, \tag{26}$$

where the letters L and R stand for the left-handed dublet ψ_L and the right-handed singlet ψ_R of one fermion generation, e.g. (ν_e, e_L^-) and e_R^-, and the operator D_μ acts on L and R as

$$D^\mu L = \left(\partial^\mu \mathbb{1}_{2x2} - ig\, T^a A^{a\mu} - ig'\,\frac{Y}{2}\mathbb{1}_{2x2} B^\mu\right) L\,; \quad D^\mu R = \left(\partial^\mu - ig'\,\frac{Y}{2} B^\mu\right) R\,. \tag{27}$$

Extract from the Lagrangian density Eq. (26) the part

$$g J_\mu^3 A^{3\mu} + \frac{1}{2} g' J_\mu^Y B^\mu \tag{28}$$

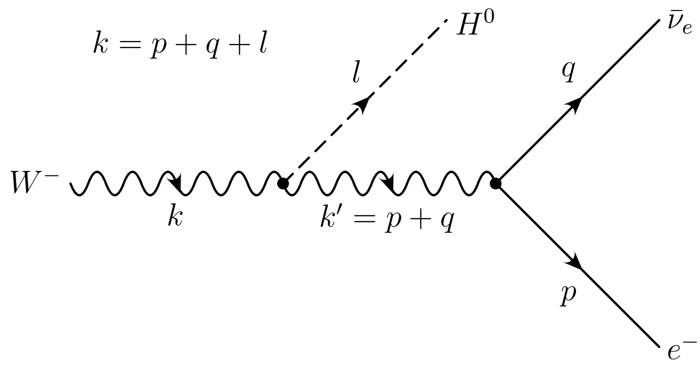
and perform the orthogonal transformation (22). Further use

$$J_\mu^{em} = J_\mu^3 + \frac{1}{2} J_\mu^Y \tag{29}$$

and show that

$$J_\mu^{NC} = J_\mu^3 - \sin^2\theta_W\, J_\mu^{em}\,. \tag{30}$$

13. Given is the decay of a real W boson, $W \to W^* + H^0 \to e\,\bar{\nu} + H^0$, where first a H^0 is radiated off.



The corresponding Lagrangians are

$$\mathcal{L}_{HWW} = g m_W\, H^0 W_\mu^+ W^{\mu-} \tag{31}$$

$$\mathcal{L}^{CC} = \frac{g}{\sqrt{2}} J_\mu^{CC} W^{\mu+} + h.c. \tag{32}$$

with $J_\mu^{CC} = \frac{1}{2}\,\bar{\nu}_e\gamma_\mu(1-\gamma_5)e$. Note that the W^+ field operator annihilates a positively charged W boson or creates a negatively charged one. Calculate the matrix element

$$\mathcal{M} = \frac{ig^2 m_W}{2\sqrt{2}}\,\frac{1}{(p+q)^2 - m_W^2}\bar{u}(p)\gamma_\mu(1-\gamma_5)v(q)\,\epsilon^\mu\,. \tag{33}$$