Furthermore, perform the summations over all spin states and the polarizations, with which you get

$$\overline{|\mathcal{M}|^2} = \frac{1}{3}\sum_{s,\lambda}|\mathcal{M}|^2 = \frac{g^4 m_W^2}{3\,[(p+q)^2 - m_W^2]^2}\left\{(p.q) + \frac{2(p.k)(q.k)}{m_W^2}\right\} \tag{34}$$

For the calculation of the decay width use the convention

$$(k-p)^2 = t\,, \quad (k-q)^2 = u\,, \quad (k-l)^2 = s\,, \tag{35}$$

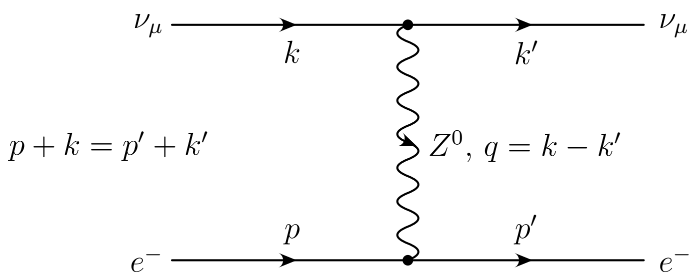
and the formula for the three-particle phase space,

$$\mathrm{dLips} = \frac{1}{128\pi^3 m_W^2}\int ds\, dt\,. \tag{36}$$

Consider the kinematic limits of the integration variables based on the results of example 15 (WS) and give the result for the differential width $\frac{\mathrm{d}^2\Gamma}{\mathrm{d}s\,\mathrm{d}t}$. Furthermore make the assumption $m_H \ll m_W$ (we know, experimentally $m_H > m_W$) and show that in this approximation the result for $\mathrm{d}\Gamma/\mathrm{d}s$ is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}s} = \frac{g^4}{768\pi^3 m_W}\left(-\frac{1}{12} + \frac{s}{12\,m_W^2} + \frac{s}{s-m_W^2}\right)\,. \tag{37}$$

14. We have the reaction $e^- + \nu_\mu \to e^- + \nu_\mu$.



Extract the necessary Feynman rules for the vertices from the Lagrangian density

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W}\, J_\mu^{NC} Z^\mu\,, \tag{38}$$

where

$$J_\mu^{NC} = \frac{1}{2}\left[g_L^f \bar{f}\gamma_\mu(1-\gamma_5)f + g_R^f \bar{f}\gamma_\mu(1+\gamma_5)f\right] \tag{39}$$

and

$$g_L^f = T_3(f_L) - Q(f_L)\sin^2\theta_W\,, \tag{40}$$
$$g_R^f = T_3(f_R) - Q(f_R)\sin^2\theta_W\,. \tag{41}$$

The Feynman rule for the Z-propagator in the unitary gauge can be found in the lecture. Calculate the matrix element, which is

$$\mathcal{M} = \frac{ig^2}{4\cos^2\theta_W} \frac{g_L^\nu}{q^2 - m_Z^2} \bar{u}(k')\gamma_\mu(1-\gamma_5)u(k) \left(\bar{u}(p')\gamma^\mu \left(g_L^e(1-\gamma_5) + g_R^e(1+\gamma_5)\right) u(p)\right) . \tag{42}$$

Further calculate the cross section $d\sigma/d\cos\theta_{CMS}$ of this reaction. The result is

$$\frac{d\sigma}{d\cos\theta_{CMS}}(e^-\nu_\mu \to e^-\nu_\mu) = \frac{g^4}{128\pi\cos^4\theta_W} \frac{(g_R^e)^2 s^2 + (g_L^e)^2 u^2}{s\,(t - m_Z^2)^2} , \tag{43}$$

with the Mandelstam variables $s = (p+k)^2$, $t = (k-k')^2$, and $u = (k-p')^2$. What is tis result for the reaction $e^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_\mu$?

15. Calculate the differential cross section of the reaction $e^+ + e^- \to b + \bar{b}$ at the Z^0 pole, i.e. the four-momentum transmitted to Z^0 is $|q| \sim m_{Z^0}$. The mass of the bottom quark is not neglected. Take the Feynman rules from the lecture notes. First show that the matrix element $\mathcal{M}$ has the form

$$\mathcal{M} = \frac{ig^2}{4\cos^2\theta_W} \frac{1}{q^2 - m_Z^2} \bar{v}(p_2)\gamma_\mu(v_e + a_e\gamma_5)u(p_1)\bar{u}(p_3)\gamma^\mu(v_b + a_b\gamma_5)v(p_4) , \tag{44}$$

and the result has the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{16\pi s} \frac{g^4}{(s - m_Z^2)^2} \left[2Am_b^2 s + (B+C)\left(s + t - m_b^2\right)^2 + (B-C)\left(m_b^2 - t\right)^2\right] , \tag{45}$$

with the factors $A = \frac{1}{16\cos^4\theta_W}(v_b^2 - a_b^2)(v_e^2 + a_e^2)$, $B = \frac{1}{16\cos^4\theta_W}(v_b^2 + a_b^2)(v_e^2 + a_e^2)$ and $C = \frac{1}{4\cos^4\theta_W} v_b a_b v_e a_e$. The Mandelstam variable s is the square of the momentum of the exchanged Z boson and t is the square of the difference of the momenta of the incoming electron and outgoing b quark. Sketch also how to obtain the total cross section.