Digital Communications 1
Written exam on June 25, 2019

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider passband PAM transmission using the QPSK signal constellation $\mathcal{A} = \{1, j, -1, -j\}$. The transmission is ISI-free with complex gain factor $p[0] = 1 - j$. The receiver performs symbolwise ML detection. The real part and the imaginary part of the noise at the slicer input, $z[k] = z_{\mathrm{R}}[k] + j z_{\mathrm{I}}[k]$, are statistically independent zero-mean Gaussian random variables with variances $\sigma_{\mathrm{R}}^2 = 0.5$ and $\sigma_{\mathrm{I}}^2 = 1$, respectively.

a) Sketch the signal constellation.

b) Sketch the signal constellation as seen at the slicer input and the decision regions of the symbolwise ML detector.

c) Assume that the symbol j was transmitted. Calculate the probabilities of detecting 1, -1, and $-j$. Calculate the conditional symbol error probability for the case that j was transmitted.

d) Calculate the conditional symbol error probabilities for the cases that 1, -1, and $-j$ were transmitted. Calculate the unconditional symbol error probability.

e) Assume that the transmitter decides to use only the symbols 1 and j but the receiver is not changed. How does this influence the unconditional symbol error probability?

Problem 2 (20 credits)

Consider a passband PAM system with received pulse

$$h(t) = e^{-t/T_s}\mathrm{u}(t) + e^{-(t/T_s-a)}\mathrm{u}(t-aT_s), \qquad \text{where } a > 0.$$

a) Specify the receive filter $f(t)$ that minimizes the noise enhancement of the ZF equalizer. Use this receive filter in the following.

b) Find a function $\phi(t)$ such that $h(t) = \phi(t) + \phi(t - aT_s)$. Calculate $r_\phi(\tau) = \int_{-\infty}^{\infty} \phi(t)\phi^*(t-\tau)dt$.

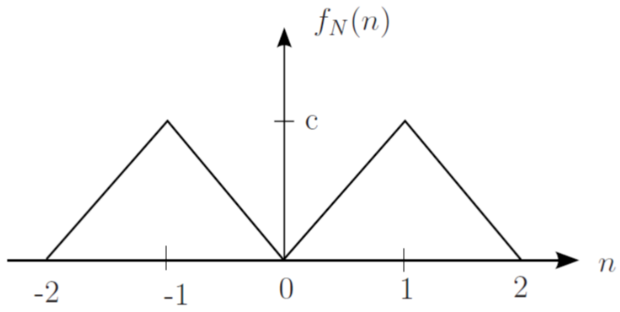
c) Calculate the equivalent discrete-time pulse $p[k]$ at the slicer input.
Hint: Use your result from b).

d) Find the value of a such that $p[0] = 2\,T_s$.

e) Assume that the transmit symbols $A[k]$ are white and equally likely with symbol alphabet $\mathcal{A} = \{1, -1\}$. Determine the value of a that maximizes the SNR at the slicer input.

Problem 3 (20 credits)

Consider transmission of a single symbol $A \in \{-1, 0\}$ over a channel with additive noise N, which is distributed as shown below.



The transmission probabilities are defined as $\mathrm{P}\{A = 0\} = p$.

a) Calculate the constant c and the MAP decision rule.

b) Calculate the ML decision rule and the corresponding symbol error probability.

c) In the following assume that the receiver knows the sign of the noise N and uses this additional information for detection.

- **c1)** Assume $N > 0$. Sketch the conditional noise pdf for $N > 0$.
- **c2)** Calculate the ML decision rule for $N > 0$ and for $N < 0$.
- **c3)** How large is the symbol error probability of the ML detector that knows the sign of N?

Problem 4 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{8z^2 + 65z + 8}{8z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of symbols $A[k] \in \{1, j, -1, -j\}$. Assume that the Viterbi algorithm is to be implemented at the output of the noise whitening filter. Sketch the state transition diagram.