Modulations- und Detektionsverfahren

Written exam on March 14, 2013

Institute of Telecommunications

Vienna University of Technology

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Two signals $x(t)$ and $y(t)$ are used for binary transmission over an AWGN channel. Both $x(t)$ and $y(t)$ are taken from the set $\{s^{(1)}(t), s^{(2)}(t), s^{(3)}(t)\}$, where

$$
\begin{aligned}
s^{(1)}(t) &= \alpha_1 \left(1 - \frac{t^2}{T^2}\right) \operatorname{rect}(t;T) \\
s^{(2)}(t) &= \alpha_2 \left(1 - \frac{t^2}{T^2}\right) \operatorname{rect}(t;T/2) \\
s^{(3)}(t) &= \alpha_3 \operatorname{rect}(t;T)\,,
\end{aligned}
$$

with $\alpha_1, \alpha_2, \alpha_3 > 0$. The receiver performs ML detection.

a) Find the values of α_1, α_2, and α_3 such that $\|s^{(1)}\|^2 = \|s^{(2)}\|^2 = \|s^{(3)}\|^2 = 1$. These values are to be used in what follows.

b) How does the error probability of the binary ML detector depend on $\|x\|^2$, $\|y\|^2$ and $\langle x, y\rangle$?

c) Find the two signals among $\{s^{(1)}(t), s^{(2)}(t), s^{(3)}(t)\}$ that minimize the error probability. These two signals are to be used as $x(t)$ and $y(t)$ in what follows.

d) One of the transmit signals ($x(t)$ or $y(t)$) is multiplied by a phase factor $e^{j\phi}$, where $\phi \in [0, 2\pi)$. Find the value of ϕ that minimizes the error probability.

Problem 2 (20 credits)

Consider transmission of a symbol $A \in \{1, 3\}$. At the slicer input, the receiver observes a random variable Q that is corrupted by real-valued multiplicative noise Z:

$$Q = A\,Z\,.$$

The noise is Laplace distributed, i.e.,

$$f_Z(z) = e^{-2\,|z-1|}\,.$$

a) For each $a \in \{1, 3\}$, calculate and sketch $f_{Q|A}(q|a)$.

b) Calculate the probabilities $\mathrm{P}\{Q > \eta_1 \,|\, A = 1\}$ and $\mathrm{P}\{Q < \eta_2 \,|\, A = 3\}$ for $\eta_1 > 1$ and $\eta_2 < 3$.

c) Consider ML detection. Calculate the decision rule and sketch the decision regions. Calculate the conditional symbol error probability $\mathrm{P}\{\mathcal{E} \,|\, A = a\}$ for each $a \in \{1, 3\}$.

d) Consider a detector using the following decision rule:

$$\hat{A} = \begin{cases} 1\,, & \big|Q-1\big| < 0.5 \\ 3\,, & \big|Q-3\big| < 0.5 \\ \text{undecided}, & \text{otherwise.} \end{cases}$$

Sketch the decision regions. Calculate the conditional symbol error probabilities $\mathrm{P}\{\hat{A} = 1 \,|\, A = 3\}$ and $\mathrm{P}\{\hat{A} = 3 \,|\, A = 1\}$.

Problem 3 (20 credits)

Consider passband PAM transmission of statistically independent symbols $a[k] \in \{0, 1, 2\}$ with equal probabilities over an AWGN channel. The receiver uses the Viterbi algorithm to implement the ML sequence detector. The impulse response of the equivalent discrete-time system including the noise whitening filter is

$$c_h[k] = 5\,\delta[k] - 4\,\delta[k-1] + \delta[k-2]\,.$$

a) Assume that a sequence of four symbols $a[1], a[2], a[3], a[4]$ is to be detected. How many possible sequences are there?

b) Let $b[k]$ denote the output of the filter (with impulse response $c_h[k]$) within the equivalent discrete-time system. Complete the tables below with the missing values of $b[k]$.

$a[k-2]$	$a[k-1]$	$a[k]$	$b[k]$
0	0	0	0
0	0	1	5
0	0	2	10
0	1	0	-4
0	1	1	1
0	1	2	6
0	2	0	-8
0	2	1	-3
0	2	2	2
1	0	0	1
1	0	1	6
1	0	2	
1	1	0	
1	1	1	2

$a[k-2]$	$a[k-1]$	$a[k]$	$b[k]$
1	1	2	7
1	2	0	-7
1	2	1	-2
1	2	2	3
2	0	0	2
2	0	1	7
2	0	2	12
2	1	0	-2
2	1	1	3
2	1	2	
2	2	0	
2	2	1	-1
2	2	2	4

c) How many symbols are contained in the state Ψ_k? How many possible states Ψ_k are there? Sketch one stage of the trellis diagram.

d) At the output of the equivalent discrete-time system, the following sequence is observed: $w[1] = 4$, $w[2] = 1$, $w[3] = 9$, $w[4] = -3$. Sketch a trellis diagram with Ψ_1, Ψ_2, Ψ_3, Ψ_4, and Ψ_5 for the case that $a[1] = 0$ and $a[2] = 2$ are known. Find the optimal path in the trellis diagram and specify $\hat{a}[3]$, $\hat{a}[4]$, and $\hat{\Psi}_1$.

Problem 4 (20 credits)

Consider the following pulse set:

$$\begin{aligned} g_1(t) &= \operatorname{sinc}\left(\pi \frac{t}{T_s}\right) \cos\left(3\pi \frac{t}{T_s}\right) \\ g_2(t) &= g_1(t/2) \\ g_3(t) &= \operatorname{sinc}\left(\frac{\pi}{2} \frac{t}{T_s}\right) \cos\left(2\pi \frac{t}{T_s}\right). \end{aligned}$$

a) Sketch the pulses in the frequency domain.

b) Which of the pulses satisfy the Nyquist criterion for symbol period T_s? (A derivation is required.)

c) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion for symbol period T_s.

d) Using the set of pulses $g_i(t)$ identified in c), consider orthogonal multipulse PAM with symbol alphabet $\mathcal{A} = \{-1, 1\}$. The symbols $A_m[k]$ are uniformly distributed and uncorrelated, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = \delta_{k,l}\delta_{m,n}$. Calculate and sketch the power spectral density $S_{\bar{S}}(j\omega)$ of the stationarized transmit signal $\bar{S}(t)$.