Modulations- und Detektionsverfahren

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Please note:

- You may use the lecture notes, a pocket calculator and a *mathematical* table of formulas of your choice.
- Personal notes, material from exercise classes, and pre-computed problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- State your derivations in detail. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A transmit symbol a from the alphabet $\mathcal{A} = \{-\frac{2d}{3}, \frac{d}{3}\}$ is transmitted with the probability $p_A(-\frac{2d}{3}) = \frac{1}{3}$ and distorted by additive noise z:

$$q = a + z.$$

The noise has a triangular distribution:

$$f_Z(z) = \frac{1}{\epsilon}\left(1 - \frac{|z|}{\epsilon}\right)\operatorname{rect}(z;\epsilon),$$

with the (initially) unknown parameter ϵ.

a) Sketch $f_Z(z)$.

b) Assume $\epsilon = \frac{d}{2}$.

b1) Sketch within one diagram $f_{Q|A}\left(q\middle| -\frac{2d}{3}\right)$ and $f_{Q|A}\left(q\middle|\frac{d}{3}\right)$, each weighted with the respective $p_A(a)$.

b2) Find the decision rule of the MAP receiver. Calculate the receiver's error probability.

c) Repeat tasks **b1)** and **b2)** assuming **(i)** $\epsilon = d$ and **(ii)** $\epsilon = 3d$.

Problem 2 (20 credits)

In a passband PAM system, the received pulse is given by

$$h(t) = \frac{1}{\sqrt{T_s}} \operatorname{rect}\left(t; \frac{T_s}{2}\right) - \frac{1}{\sqrt{T_s}} \operatorname{rect}\left(t - \frac{3}{4}T_s; \frac{T_s}{4}\right).$$

a) Calculate the impulse response $\rho_h[k]$ and the transfer function $S_h(z)$ of the equivalent dicrete-time system (including the sampled matched filter).

b) Find the zeros and poles of $S_h(z)$.

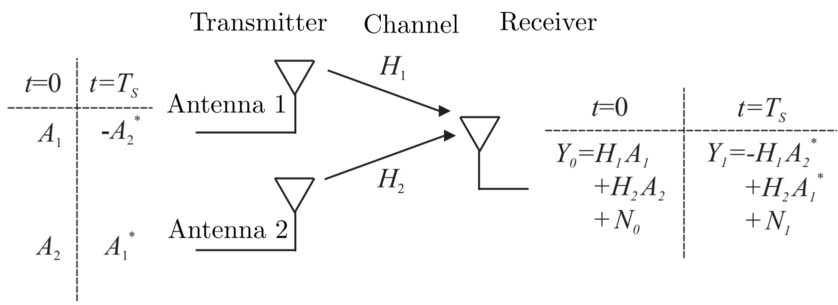
c) Calculate the linear *zero forcing* equalizer and find its poles.

d) Find a minimum-phase factorization of $S_h(z)$.

e) Calculate the transfer function and the impulse response of the equivalent discrete-time system including the *noise-whitening* filter.

Problem 3 (20 credits)

Let us examine an orthogonal space-time code (a so-called "Alamouti code") for two transmit antennas and one receive antenna:



Consider two time instants (t=0, t=T_s):

- Antenna 1 transmits the symbols A_1 $(t = 0)$ and $-A_2^*$ $(t = T_s)$.
- Antenna 2 transmits the symbols A_2 $(t = 0)$ and A_1^* $(t = T_s)$.

The transmit symbols $A_1, A_2 \in \{1, j, -1, -j\}$ are equally likely and statistically independent. H_1 and H_2 are the channel fading coefficients between the antennas. Y_i denotes the received signal and N_i the additive noise at time instant i. All variables are complex-valued. The noise is circularly Gaussian distributed with mean $\mathrm{E}\{\mathbf{n}\} = \mathbf{0}$ and correlation matrix $\mathbf{R_n} = \mathrm{E}\{\mathbf{n}\,\mathbf{n}^H\} = \sigma_n^2\,\mathbf{I}$, where $\mathbf{n} = (N_0\ N_1)^T$.

a) What is the bit rate R_b of the transmission? ($T_s = 0.5$ms)

b) The input-output relation of the transmission system can be expressed as follows (see figure above):

$$\underbrace{\begin{pmatrix} Y_0 \\ Y_1^* \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}}_{\mathbf{H}} \underbrace{\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}}_{\mathbf{a}} + \underbrace{\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}}_{\mathbf{w}}. \tag{1}$$

Specify $\mathbf{H}$ and $\mathbf{w}$.

c) Calculate the receive vector after the "matched filter"

$$\mathbf{z} = \mathbf{H}^H\,\mathbf{y} \tag{2}$$

as a function of $\mathbf{a}$ and $\mathbf{w}$. Simplify $\mathbf{H}^H\mathbf{H}$ as far as possible.

d) Determine the mean and variance of the noise after the matched filter. Find the correlation matrix of the filtered noise. Are the two noise components still statistically independent after filtering?

e) Design an ML receiver for A_1 and A_2 and find an *exact* expression for the respective symbol error probabilities.

Problem 4 (20 credits)

A transmission system with a distortion-free AWGN channel with noise power spectral density $N_0/2 = 2 \cdot 10^{-6}\mathrm{W/Hz}$ uses the following four transmit signals:

$$\begin{aligned} s^{(1)}(t) &= \quad A\cos\left(\tfrac{\pi}{T}t\right)\mathrm{rect}\left(t;\tfrac{T}{2}\right) \\ s^{(2)}(t) &= -\,A\cos\left(\tfrac{\pi}{T}t\right)\mathrm{rect}\left(t;\tfrac{T}{2}\right) \\ s^{(3)}(t) &= \quad A\cos\left(\tfrac{\pi}{T}t\right)\mathrm{rect}\left(t;\tfrac{T}{2}\right)\mathrm{sgn}(t) \\ s^{(4)}(t) &= -\,A\cos\left(\tfrac{\pi}{T}t\right)\mathrm{rect}\left(t;\tfrac{T}{2}\right)\mathrm{sgn}(t) \end{aligned} \qquad \text{with } \mathrm{sgn}(t) = \begin{cases} 1, & t \geq 0 \\ -1, & t < 0, \end{cases}$$

where $A^2 = 0.8\mathrm{W}$, $T = 5 \cdot 10^{-5}\mathrm{s}$.

a) Sketch the four transmit signals.

b) The transmit signals are to be expressed using an orthogonal (not necessarily orthonormal) basis.

b1) $s^{(1)}(t)$ is to be used as the first basis function. Which of the transmit signals can be used as the second basis function, which cannot?

b2) Choose one of the transmit signals as the second basis function and normalize the two basis functions. Express the four transmit signals as coefficient vectors. Find the variances of the noise coefficients.

b3) Sketch the location of the transmit signals as points in the two-dimensional vector space.

c) Assume the following transmit probabilities: $p_I(1) = 1/3$, $p_I(2) = 1/6$, $p_I(3) = 1/3$, $p_I(4) = 1/6$.

c1) Sketch the decision regions of the ML sequence detector.

c2) Calculate the error probability of the ML sequence detector.

c3) Sketch the decision regions of the MAP sequence detector.