Digital Communications 1
Written exam on June 28, 2021

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Binary symbols $a[k] \in \{1, -1\}$ are transmitted by means of passband PAM over a dispersive channel with additive noise. The transmit symbols are equally likely. The equivalent baseband overall pulse is

$$p(t) = \alpha \left(1 - \frac{t^2}{\tau^2}\right) \mathrm{rect}\left(t; \frac{4\tau}{3}\right), \qquad \text{with } \tau = 10\text{ms},\ \alpha > 0.$$

Consider a simple receiver whose symbol-rate sampler is followed directly by a slicer.

a) Sketch the overall pulse $p(t)$. Determine the maximum bit rate for ISI-free transmission and the corresponding symbol duration T_s. This bit rate is to be used in what follows.

b) Sketch the transmission system in the equivalent discrete-time baseband domain.

c) How does the output $q[k]$ of the symbol-rate sampler depend on the transmit symbols $a[k]$?

d) At the slicer input, the filtered discrete-time noise is zero-mean, white, and complex Gaussian with variance N_0. The slicer uses the following decision rule:

$$\hat{a}[k] = \begin{cases} 1 & \text{if } \mathrm{Re}\{q[k]\} > 0, \\ -1 & \text{if } \mathrm{Re}\{q[k]\} < 0. \end{cases}$$

What is the symbol error probability of the receiver?

Problem 2 (20 credits)

Consider equalization of a channel using a decision feedback equalizer. The equivalent discrete-time baseband pulse is given by

$$p[k] = \delta[k] + \frac{1}{2}\delta[k-1] - \frac{1}{2}\delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1/4$.

a) Assume a general feedforward filter $d[k]$, $k \in [-L, L]$. Calculate the equivalent overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

b) Consider now the case $L = 1$. What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous symbol decisions were correct)?

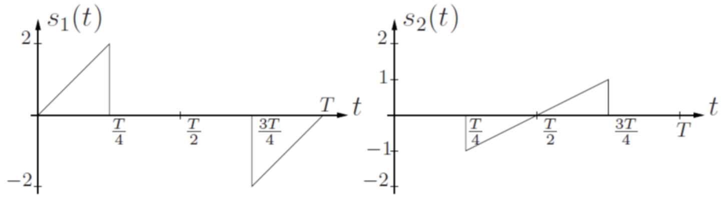
c) The coefficients of the feedforward filter are $\mathbf{d}_{\text{MSE}} = \frac{2}{25}(3, 7, -4)^T$. Calculate the coefficients of the feedback filter (of length K, as calculated in b)) that minimize the MSE at the slicer input.

d) Specify the signal at the slicer input and indicate the components that constitute 1.) the desired signal, 2.) ISI, and 3.) additive noise (still assuming that all previous symbol decisions were correct).

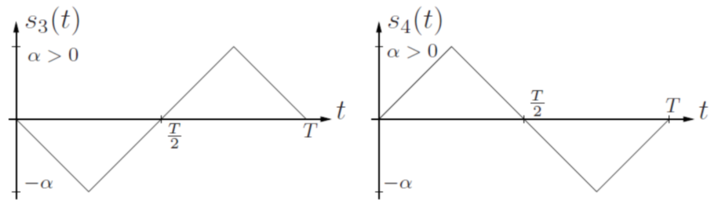
e) Calculate the mean powers of the three signal components from d).

Problem 3 (20 credits)

For the transmission of a binary symbol (both values are equally likely) over a channel corrupted by additive white Gaussian noise with power spectral density $N_0/2$, two different signals are used. Let us compare the following alternative signal sets:



Signal set I



Signal set II

a) Represent all four signals in terms of a common orthonormal basis that is as simple and low-dimensional as possible. Specify that basis.

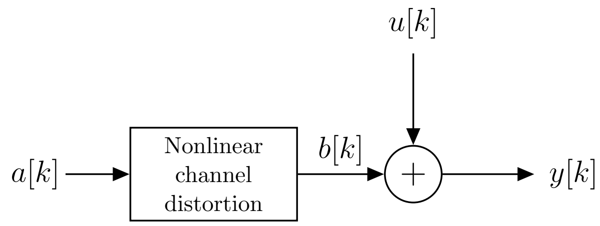
b) Calculate the amplitude $\alpha > 0$ for signal set II such that the average signal energy is equal for both signal sets.

c) For the detection of the transmitted binary symbol, an optimum receiver is employed. Calculate the error probabilities obtained with signal set I and signal set II using the amplitude α determined in Part b).

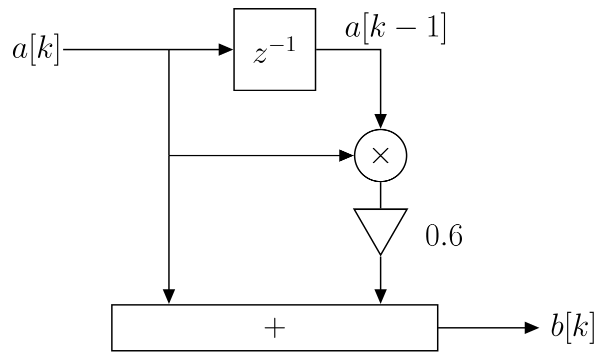
d) Calculate the amplitude $\alpha > 0$ such that the error probability obtained with signal set II is equal to the error probability obtained with signal set I.

Problem 4 (20 credits)

A sequence of binary symbols $a[k] \in \{0, 1\}$ is transmitted over a discrete-time, nonlinear channel with additive white Gaussian noise $u[k]$. The discrete-time baseband system is given by the following block diagram:



where the nonlinear distortion introduced by the channel is defined by the following shift register circuit:



a) Provide the input-output relation describing the dependence of the output sequence $b[k]$ on the input sequence $a[k]$.

b) Sketch the state diagram and one stage of the trellis diagram. Label the branches with the input-output pairs $(a[k];\, b[k])$. How many different states are there?

c) Use the Viterbi algorithm to identify the path with minimum distance from the zero path $a[k] \equiv 0$. You may assume that the initial state is zero. Determine the corresponding symbol sequence, the distance from the zero path, and the number of symbol errors.

d) Determine the symbol sequence obtained with the ML sequence detector for the received sequence $(y[k]) = \cdots 0, 0, 1, 0, 0 \cdots$ and calculate the corresponding path metric.