Digital Communications 1

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider replacing the zero-forcing equalizer

$$D_{\mathrm{ZF}}(e^{j\theta}) = \frac{1}{1 - 2\cos\theta}$$

by a decision feedback equalizer. The transmit symbols of the system are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1$.

a) Calculate the pulse $p[k]$ of the equivalent discrete-time baseband channel consisting of transmit pulse, physical channel, and matched receive filter.

b) Assume a general feedforward filter $d[k], k \in [-L, L]$. Calculate the overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

c) Consider now the case $L = 1$. What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous symbol decisions were correct)?

d) Calculate the coefficients of the MSE-optimum feedforward filter $\mathbf{d}_{\mathrm{MSE}}$.

Hint: The inverse of a matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

can be calculated as

$$\mathbf{A}^{-1} = \frac{1}{\det\mathbf{A}} \begin{pmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{13}a_{32} - a_{12}a_{33} & a_{12}a_{23} - a_{13}a_{22} \\ a_{23}a_{31} - a_{21}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{13}a_{21} - a_{11}a_{23} \\ a_{21}a_{32} - a_{22}a_{31} & a_{12}a_{31} - a_{11}a_{32} & a_{11}a_{22} - a_{12}a_{21} \end{pmatrix},$$

with

$$\det\mathbf{A} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

Problem 2 (20 credits)

A binary random variable $S \in \{-1, 1\}$ with $p_S(1) = 1/3$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & n \geq b \\ 0 & n < b, \end{cases}$$

with a given $b \in \mathbb{R}$.

a) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

b) Calculate the parameter $b \in \mathbb{R}$ such that $\mathrm{P}\{N > 1\} = 1/2$. (This value of b is to be used in what follows.)

c) Calculate and sketch the ML decision rule.

d) Calculate and sketch the MAP decision rule.

Problem 3 (20 credits)

Consider transmission of the signals

$$s^{(i)}(t) = i^2 s(t), \quad i \in \{1, 2, 3\} \quad \text{with } p_I(1) = p_I(2)/2 = 1/5$$

over an AWGN channel with noise variance $N_0/2$. Here, $s(t)$ is a fixed signal with energy E_s.

a) Determine an orthonormal basis of the transmit signal space.

b) Specify the vectors corresponding to the signals $s^{(i)}(t)$.

c) Sketch an implementation of the ML detector using inner products of the received signal with the basis functions.

d) Sketch an implementation of the ML detector using inner products of the received signal with the transmit signals.

e) Calculate the conditional sequence error probabilities and the unconditional sequence error probability of the ML detector.

Problem 4 (20 credits)

Consider a bandpass PAM system with symbol alphabet $\mathcal{A} = \{-3, 1, -j\}$; the symbols $A[k]$ are assumed statistically independent and uniformly distributed.

a) Calculate the symbol power P_A. How should the symbol alphabet be shifted in order to achieve minimum symbol power?

The shifted symbol alphabet is to be used in what follows.

b) For transmission over a data link, the symbol sequence $A[k]$ is transformed into the sequence $B[k] = A[k] - \alpha A[k-3]$ with $\alpha \in \mathbb{R}$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transformed symbols $B[k]$.

c) The transmit signal in the equivalent baseband is $S_{\text{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\text{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\text{LP}}(t)$.

d) Choose α such that the power spectral density $S_{\bar{S}_{\text{LP}}}(j\omega)$ is zero at frequency $\omega = \frac{\pi}{2T_s}$.