Digital Communications 1
Written exam on October 19, 2016

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider passband PAM transmission using the QPSK signal constellation $\mathcal{A} = \{1, j, -1, -j\}$. The transmission is ISI-free with complex gain factor $p[0] = 1 + j$. The receiver performs symbolwise ML detection. The real part and the imaginary part of the noise at the slicer input, $z[k] = z_{\mathrm{R}}[k] + j z_{\mathrm{I}}[k]$, are statistically independent zero-mean Gaussian random variables with variances $\sigma_{\mathrm{R}}^2 = 1$ and $\sigma_{\mathrm{I}}^2 = 0.25$, respectively.

a) Sketch the signal constellation.

b) Sketch the signal constellation as seen at the slicer input and the decision regions of the symbolwise ML detector.

c) Assume that the symbol -1 was transmitted. Calculate the probabilities of detecting 1, j, and $-j$. Calculate the conditional symbol error probability for the case that -1 was transmitted.

d) Calculate the conditional symbol error probabilities for the cases that 1, j, and $-j$ were transmitted. Calculate the unconditional symbol error probability.

e) Assume that the transmitter decides to use only the symbols 1 and -1 but the receiver is not changed. How does this influence the unconditional symbol error probability?

Problem 2 (20 credits)

A binary random variable $S \in \{-1, 1\}$ with $p_S(1) = 1/3$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & \text{if } n \geq b, \\ 0 & \text{if } n < b, \end{cases}$$

with a given $b \in \mathbb{R}$.

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

c) Calculate the parameter $b \in \mathbb{R}$ such that $\mathrm{P}\{N > 1\} = 1/2$. (This value of b is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.

Problem 3 (20 credits)

In a passband PAM system with symbol period T_s, the received pulse is given by

$$h(t) = \sum_{n=0}^{\infty} 2^{-n}\, e^{-\alpha(t-n\,T_s)}\, \mathrm{rect}\Big(t - n\,T_s, \frac{T_s}{2}\Big), \qquad \alpha > 0.$$

a) Calculate the impulse response $\rho_h[k]$ of the equivalent discrete-time system (including the sampled matched filter).

b) Calculate the transfer function $S_h(z)$ and sketch $S_h(e^{j\theta})$.

c) Find the zeros and poles of $S_h(z)$ and sketch them in the complex plane.

d) Calculate the linear zero forcing equalizer and find its poles.

e) Find the minimum-phase factorization of $S_h(z)$.

f) Calculate the transfer function and impulse response of the equivalent discrete-time system that includes the noise-whitening filter corresponding to the result of part e).

Problem 4 (20 credits)

Consider an orthogonal multipulse PAM system with M transmit pulses $g_m(t)$ and random transmit symbols $A_m[k]$, $m \in \{1, \ldots, M\}$, $k \in \mathbb{Z}$.

a) Derive an expression of the autocorrelation function $R_{\bar{S}}(\tau)$ of the stationarized transmit signal $\bar{S}(t)$ without making any assumptions about the symbol statistics.

b) Simplify $R_{\bar{S}}(\tau)$ from a) for the case that the symbols $A_m[k]$ are zero-mean and uncorrelated with respect to m, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = R_A^{(m)}[k-l]\delta_{m,n}$. Calculate the power spectral density $S_{\bar{S}}(j\omega)$.

c) Further specialize $R_{\bar{S}}(\tau)$ and $S_{\bar{S}}(j\omega)$ from b) for the case where the symbols $A_m[k]$ are uncorrelated with respect to k and have constant (but still m-dependent) variances $\sigma_{A_m}^2$.

d) Assume transmit pulses $g_m(t) = G\cos\big((2m-1)\omega_0 t\big)\,\mathrm{sinc}\big(\pi\frac{t}{T_s}\big)$, $m \in \{1, \ldots, M\}$, where $\omega_0 = 2\pi/T_s$. Calculate and sketch $S_{\bar{S}}(j\omega)$ for zero-mean, uncorrelated and uniformly distributed symbols $A_m[k] \in \{-2, 2\}$.

e) Determine the transmit bandwidth and spectral efficiency if the transmit pulses from d) are used.