Digital Communications 1
Written exam on September 22, 2020

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider passband PAM transmission using the signal constellation

$$\mathcal{A} = \{-1 \pm j, 1 \pm j, \beta \pm j\},$$

with $\beta > 1$. The receiver performs symbolwise ML detection. The symbols $a[k] \in \mathcal{A}$ are corrupted by additive noise $z[k] \in \mathbb{C}$, yielding the following signal at the slicer input:

$$q[k] = a[k] + z[k],$$

with real part $q_{\mathrm{R}}[k]$ and imaginary part $q_{\mathrm{I}}[k]$. The real part and the imaginary part of the noise at the slicer input, $z[k] = z_{\mathrm{R}}[k] + j z_{\mathrm{I}}[k]$, are statistically independent zero-mean Gaussian random variables with variances $\sigma_{\mathrm{R}}^2 = 1$ and $\sigma_{\mathrm{I}}^2 = 0.5$, respectively.

a) Sketch the signal constellation and the decision regions of the symbolwise ML detector.

b) Assume that the symbol $1 - j$ was transmitted. Specify the ML decision region for a correct decision by specifying the intervals within which $q_{\mathrm{R}}[k]$ and $q_{\mathrm{I}}[k]$ must lie.

c) Calculate the probability of a correct decision for the case that $1 + j$ was transmitted, depending on β. Find the value of β for which this probability is 0.6.

d) The prior probabilities of symbols with a positive real part is twice as large as that of symbols with a negative real part. Using the value of β calculated in part c), calculate the mean symbol power P_A.

Problem 2 (20 credits)

Consider a passband PAM system with symbol period T_s, transmit pulse $g(t)$ with energy $E_g = 1$, and transmit symbols $A[k]$. The receive pulse in the equivalent baseband is $h(t) = g(t)+\alpha g(t-2T_s)$ with $|\alpha| < 1$. The additive channel noise has a power spectral density of $N_0/2$. The receiver consists of a sampled matched filter, a noise whitening filter (based on a *causal* factorization), a slicer, and an equalizer.

a) Determine the impulse response, transfer function, and noise power spectral density of the equivalent discrete-time model of the transmission system including the noise whitening filter but not the equalizer.

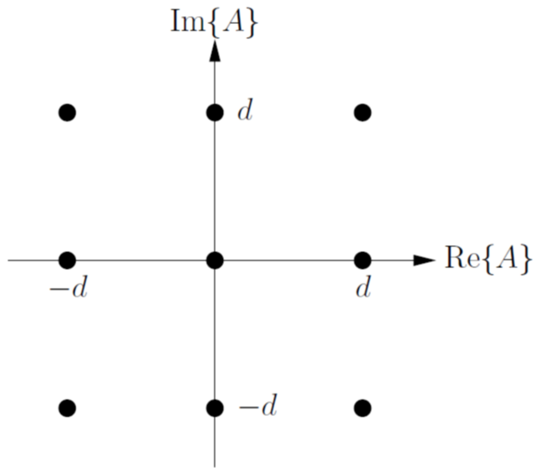
b) Calculate the zero-forcing equalizer (placed after the noise whitening filter). Compare the noise variance at the input and the output of the equalizer.

c) Verify that the output of the noise whitening filter is free of precursor intersymbol interference.

d) Determine the decision-feedback equalizer that completely removes postcursor intersymbol interference. (The noise whitening filter is used as the feedforward filter of the decision-feedback equalizer.)

Problem 3 (20 credits)

Consider the transmission of a single symbol A from the following signal constellation:



The transmit probabilities are

$$\mathrm{P}\{A = \pm d\} = \mathrm{P}\{A = \pm jd\} = 1/10 \quad \text{and} \quad \mathrm{P}\{A = (-1 \pm j)d\} = \mathrm{P}\{A = (1 \pm j)d\} = 1/20.$$

Note that the transmit probability $\mathrm{P}\{A = 0\}$ was not specified and must be deduced. The received symbol is $Q = A + Z$, where Z is circularly symmetric complex Gaussian noise with variance $\sigma_Z^2 = d^2/10$. Symbol A and noise Z are statistically independent.

a) Calculate the symbol mean μ_A and the symbol variance σ_A^2.

b) Calculate and sketch the decision regions of the MAP receiver.

c) Calculate the error probability of the MAP receiver.

d) Calculate the error probability of the ML receiver.

Problem 4 (20 credits)

Consider a variant of FSK using the transmit pulses

$$g_m(t) = G\cos\big((2m+1)\omega_0 t\big)\,\mathrm{sinc}(\omega_0 t), \qquad m = 1, \dots, M_a$$

where $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$ and $\omega_0 = \frac{\pi}{T_s}$.

a) Calculate and sketch the Fourier transform $G_m(j\omega)$.

b) Show that all shifted transmit pulses $g_m(t - kT_s)$ and $g_n(t - lT_s)$ $(k, l \in \mathbb{Z})$ are orthogonal unless both $k = l$ and $n = m$.

c) Calculate and sketch the transmit bandwidth B_T and the spectral efficiency ν as a function of M_a.