Modulations- und Detektionsverfahren

Written exam on June 25, 2013

Institute of Telecommunications

Vienna University of Technology

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider the transmission of a symbol $A \in \{-1, 1\}$ over a single-input, multiple-output (SIMO) system using one transmit antenna and two receive antennas. The input-output relation of the SIMO system is

$$\mathbf{y} = A\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} = (Y_1\ Y_2)^{\mathrm{T}}$ denotes the received vector, $\mathbf{h} = (h_1\ h_2)^{\mathrm{T}}$ describes the SIMO channel, and $\mathbf{n} = (N_1\, N_2)^{\mathrm{T}}$ is the additive noise. All variables are real-valued. The noise is jointly Gaussian distributed with mean $\mathrm{E}\{\mathbf{n}\} = \mathbf{0}$ and correlation matrix $\mathbf{R_n} = \mathrm{E}\{\mathbf{n}\,\mathbf{n}^T\} = \sigma_n^2\,\mathbf{I}$. The channel fading coefficients are $h_1 = 1$ and $h_2 = 1/2$.

a) For each $a \in \{-1, 1\}$, calculate and sketch $f(\mathbf{y}|a)$ in the $(y_1,\, y_2)$ plane.

b) Consider ML detection of A. Calculate the ML decision rule and sketch the corresponding decision regions.

c) Consider MAP detection of A, assuming that the prior distribution of A is given by

$$p_A(a) = \begin{cases} 2/3, & a = -1 \\ 1/3, & a = 1\,. \end{cases}$$

Calculate the posterior probability of each $A \in \{-1, 1\}$ for $y_1 = 0.5$, $y_2 = -0.5$, and $\sigma_n^2 = 0.5$. What is $\hat{A}_{\mathrm{MAP}}$ in this case?

d) Consider an alternative receiver using a "matched filter" whose output is

$$Z = \mathbf{h}^T\,\mathbf{y}.$$

d1) Express Z as a function of A, N_1, and N_2. Calculate $f_{Z|A}(z|a)$.

d2) Find the decision rule of the ML detector using Z.

Problem 2 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{5z^2 + 26z + 5}{5z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of statistically independent symbols $a[k] \in \{1, j, -1, -j\}$ with equal probabilities. Assume that the Viterbi algorithm is used at the output of the noise whitening filter. Sketch the state transition diagram.

Problem 3 (20 credits)

Consider transmission of a symbol $A \in \{-1, 1\}$. At the slicer input, the receiver observes a random variable Q that is corrupted by real-valued additive noise Z:

$$Q = A + Z\,.$$

The noise is exponentially distributed, i.e.,

$$f_Z(z) = \begin{cases} e^{-z} & z \geq 0 \\ 0 & z < 0\,. \end{cases}$$

a) For each $a \in \{-1, 1\}$, calculate and sketch $f_{Q|A}(q|a)$.

b) Calculate the probabilities $\mathrm{P}\{Q > \eta_1 \,|\, A = -1\}$ and $\mathrm{P}\{Q < \eta_2 \,|\, A = 1\}$ for $\eta_1 > -1$ and $\eta_2 < 1$.

c) Consider ML detection. Determine the decision rule and sketch the decision intervals. Calculate the conditional symbol error probability $\mathrm{P}\{\mathcal{E} \,|\, A = a\}$ for each $a \in \{-1, 1\}$.

d) Consider a detector using the following decision rule:

$$\hat{A} = \begin{cases} -1\,, & -1 \leq Q \leq 0 \\ 1\,, & 1 \leq Q \leq 2 \\ \text{undecided}, & \text{otherwise.} \end{cases}$$

Sketch the decision intervals. Calculate the conditional symbol error probabilities $\mathrm{P}\{\hat{A} = 1 \,|\, A = -1\}$ and $\mathrm{P}\{\hat{A} = -1 \,|\, A = 1\}$.

Problem 4 (20 credits)

Consider equalization of a channel using a decision feedback equalizer. The equivalent discrete-time baseband pulse is given by

$$p[k] = \delta[k] - \frac{1}{4}\delta[k-1] - \frac{1}{4}\delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1/2$.

a) Assume a general feedforward filter $d[k], k \in [-L, L]$. Calculate the equivalent overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

b) Consider now the case $L = 1$. What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous symbol decisions were correct)?

c) The coefficients of the feedforward filter are $\mathbf{d}_{\text{MSE}} = (1/163) \cdot (7\ 104\ -2)^T$. Calculate the coefficients of the feedback filter (of length K, as calculated in b)) that minimize the MSE at the slicer input.

d) Specify the signal at the slicer input and indicate the components that constitute 1.) the desired signal, 2.) ISI, and 3.) additive noise (still assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).