Modulations- und Detektionsverfahren

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, material from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A transmittor chooses from 4 signals represented by the following vectors (via an orthonormal basis expansion):

$$\mathbf{s}^{(1)} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \quad \mathbf{s}^{(2)} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \quad \mathbf{s}^{(3)} = \begin{pmatrix} -c \\ 0 \end{pmatrix}, \quad \mathbf{s}^{(4)} = \begin{pmatrix} 0 \\ -c \end{pmatrix},$$

with $c^2 = 10$ mJ. The transmit probability $p_I(1) = p_I(2)$ is 9 times as large as $p_I(3) = p_I(4)$. The signal is transmitted over an AWGN channel with $N_0/2 = 2 \cdot 10^{-4}$ W/Hz.

a) Consider the ML detector. Calculate its decision rule and sketch the decision regions. Calculate the resulting error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$.

b) Consider the MAP detector. Calculate its decision rule and sketch the decision regions. Calculate the resulting error probability $\mathrm{P}_{\mathrm{MAP}}\{\mathcal{E}\}$.

c) Reconsider the ML detector. By what factor must c be changed to achieve the error probability calculated for the MAP detector in b)? (*Hint:* Use the expression for $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ in which c appears only once.)

d) The entire symbol alphabet is shifted by adding a vector $\mathbf{d}$. Calculate the $\mathbf{d}$ that minimizes the transmit signal power **(i)** for the original signal constellation and **(ii)** for the scaled signal constellation calculated in c). How do these shifts affect the error probabilities $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ and $\mathrm{P}_{\mathrm{MAP}}\{\mathcal{E}\}$?

Problem 2 (20 credits)

In a passband PAM system, the received pulse is given by

$$h(t) = \frac{1}{\sqrt{T_s}}\, e^{-|t|/T_s}\, \mathrm{rect}(t; T_s).$$

a) Calculate the impulse response $\rho_h[k]$ and the transfer function $S_h(z)$ of the equivalent dicrete-time system (including the sampled matched filter).

b) Find the zeros and poles of $S_h(z)$.

c) Find a minimum-phase factorization of $S_h(z)$.

d) Calculate the transfer function and the impulse response of the equivalent discrete-time system including the noise-whitening filter.

Problem 3 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-2, 2, -2j, 2j\}$. The symbols $A[k]$ are assumed white and uniformly distributed.

a) For transmission, the symbol sequence $A[k]$ is transformed into a sequence $B[k] = A[k] + \alpha A[k-1]$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transformed symbols $B[k]$.

b) The equivalent baseband transmit signal is $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k] g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

c) The spectrum $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is required to be zero at frequency $\omega = \frac{\pi}{T_s}$. Which choice of α satisfies this condition?

d) Assume that the transmit pulse $g(t)$ is a sinc pulse: $g(t) = \mathrm{sinc}(\frac{\pi t}{T_s})$. Sketch the power spectral density of the stationarized transmit signal $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ for this case, using the α calculated in c).

Problem 4 (20 credits)

For two complex signals $x(t)$ and $y(t)$, consider the quantities $\|x\|$, $\|y\|$, $\langle x, y\rangle$, and $\|x-y\|$.

a) Show how these quantities are changed by the following transformations of *both* signals $x(t)$ and $y(t)$:

a1) a time shift by t_0,

a2) a frequency shift by ω_0,

a3) multiplication by a constant factor $\alpha \in \mathbb{C}$,

a4) time scaling according to $x(t) \to \sqrt{|a|}\, x(at)$ and $y(t) \to \sqrt{|a|}\, y(at)$, with $a \in \mathbb{R}\backslash\{0\}$. *Hint*: use $a = |a| \cdot \operatorname{sign}(a)$.

b) Consider the signals

$$x(t) = A\cos(\omega_0 t)\ \operatorname{rect}(t; T/2), \qquad y(t) = A\cos(\omega_0 t + \theta)\ \operatorname{rect}(t; T/2),$$

with $T = 2\pi/\omega_0$.

b1) Calculate $\|x\|$, $\|y\|$, $\langle x, y\rangle$, and $\|x-y\|$.

b2) Sketch these quantities as a function of θ.