Digital Communications 1
Written exam on June 23, 2015

Institute of Telecommunications

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Please note:

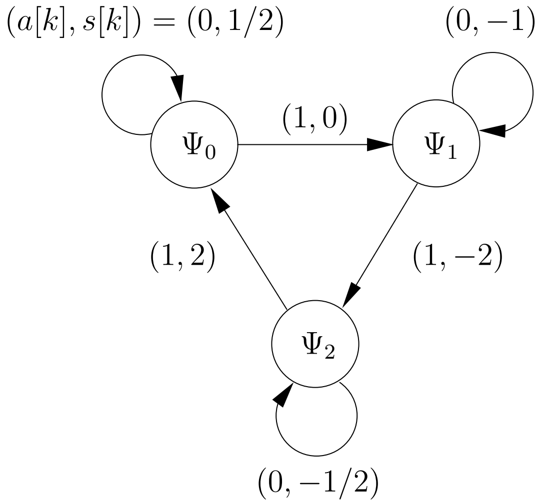
- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

In a discrete-time transmission system, the sequence

$$y[k] = s[k] + n[k], \qquad k = 1, \ldots, K \tag{1}$$

is received. Here, the signal sequence $s[k] \in \{-2, -1, -1/2, 0, 1/2, 2\}$ is obtained from the sequence of transmit symbols $a[k] \in \{0, 1\}$ according to the state diagram shown below. The noise values $n[k]$ are statistically independent across k and statistically independent of the symbol sequence $a[k]$.



a) Assume that the noise is Cauchy-distributed, i.e,

$$f_N(n) = \frac{1}{\pi} \frac{1}{1 + n^2}.$$

a1) Determine the corresponding ML sequence detector.

a2) Show that it is possible to implement this ML sequence detector by using the Viterbi algorithm and a suitable branch metric. Sketch one stage of the trellis diagram.

a3) Determine the symbol sequence $\hat{a}[k]$ detected by the ML sequence detector if the sequence $y[0] = 0.1$, $y[1] = -5$, $y[2] = 0.6$ has been received. Assume that at $k = 0$ the state Ψ_0 is in force.

b) For $y[0]$, $y[1]$, and $y[2]$ as in a3), determine the symbol sequence $\hat{a}[k]$ that is produced by the ML sequence detector designed for Gaussian noise.

Problem 2 (20 credits)

Consider a variant of FSK using the transmit pulses

$$g_m(t) = G\,\cos((2m+1)\omega_0 t)\,\mathrm{sinc}(\omega_0 t), \qquad m = 1, \ldots, M_a, \qquad \omega_0 = \frac{\pi}{T_s},$$

where $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$.

a) Calculate and sketch the Fourier transform $G_m(j\omega)$.

b) Show that all shifted transmit pulses $g_m(t - kT_s)$ and $g_n(t - lT_s)$ $(k, l \in \mathbb{Z})$ are orthogonal unless both $k = l$ and $n = m$.

c) Calculate and sketch the transmit bandwidth B_T and the spectral efficiency ν as a function of M_a.

Problem 3 (20 credits)

A transmission system uses the two transmit signals

$$s_1(t) = \sin(\omega_0 t)\ \mathrm{rect}(t - T; T)$$
$$s_2(t) = \sin(\omega_0 t - \theta)\ \mathrm{rect}(t - T; T),$$

with $T = \pi/\omega_0$ and $\omega_0 > 0$.

a) Calculate $\langle s_1, s_2 \rangle$ and sketch it as a function of θ.

b) Calculate $\|s_1\|$, $\|s_2\|$, and $\|s_1 - s_2\|$.

Consider transmission of $s_i(t)$, $i \in \{1, 2\}$ over an AWGN channel:

$$y(t) = s_i(t) + n(t),$$

where the noise $n(t)$ has a power spectral density of $N_0/2$.

c) Find the decision rule of the ML detector $\hat{\imath}_{\mathrm{ML}}(y)$. Sketch an implementation of the ML detector that uses only one inner product.

d) Using your results from b), calculate the error probability of the ML detector.

Problem 4 (20 credits)

Consider transmission of a symbol $A \in \{-1/2, 1\}$. At the slicer input, the receiver observes a random variable Q that is corrupted by real-valued additive noise Z:

$$Q = A + Z\,.$$

The noise is exponentially distributed, i.e.,

$$f_Z(z) = \begin{cases} e^{-z} & z \geq 0 \\ 0 & z < 0\,. \end{cases}$$

a) For each $a \in \{-1/2, 1\}$, calculate and sketch $f_{Q|A}(q|a)$.

b) Calculate the probabilities $\mathrm{P}\{Q > \eta_1 \,|\, A = -1/2\}$ and $\mathrm{P}\{Q < \eta_2 \,|\, A = 1\}$ for $\eta_1 > -1/2$ and $\eta_2 < 1$.

c) Consider ML detection. Determine the decision rule and sketch the decision intervals. Calculate the conditional symbol error probability $\mathrm{P}\{\mathcal{E} \,|\, A = a\}$ for each $a \in \{-1/2, 1\}$.

d) Consider a detector using the following decision rule:

$$\hat{A} = \begin{cases} -1/2\,, & -1/2 \leq Q \leq 0 \\ 1\,, & 1 \leq Q \leq 2 \\ \text{undecided}, & \text{otherwise.} \end{cases}$$

Sketch the decision intervals. Calculate the conditional symbol error probabilities $\mathrm{P}\{\hat{A} = 1 \,|\, A = -1/2\}$ and $\mathrm{P}\{\hat{A} = -1/2 \,|\, A = 1\}$.