Modulations- und Detektionsverfahren

Written exam on May 17, 2010

Institute of Communications an Radio Frequency Engineering

Vienna University of Technology

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, material from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A binary random variable $S \in \{-2, 1\}$ (with $p_S(-2) = 1/3$) is corrupted by additive noise N that is statistically independent of S and has a shifted Laplacian distribution

$$f_N(n) = a\, e^{-\lambda|n-1|}.$$

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

c) Calculate the parameter λ such that $\mathrm{P}\{N < 0\} = 1/8$. (This value of λ is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.

Problem 2 (20 credits)

Consider equalization of a channel using a decision feedback equalizer. The equivalent discrete-time baseband pulse is given by

$$p[k] = \delta[k] + \frac{1}{2}\delta[k-1] - \frac{1}{2}\delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1/4$.

a) Assume a general feedforward filter $d[k], k \in [-L, L]$. Calculate the equivalent overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

b) Consider now the case $L = 1$. What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous symbol decisions were correct)?

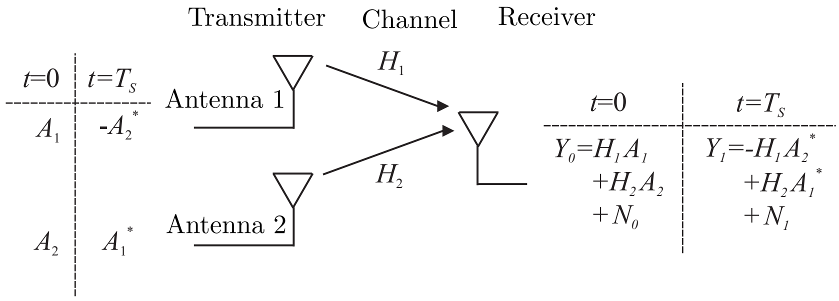
c) The coefficients of the feedforward filter are $\mathbf{d}_{\text{MSE}} = (2/25)\cdot(3\ 7\ -4)^T$. Calculate the coefficients of the feedback filter (of length K, as calculated in b)) which minimize the MSE at the slicer input.

d) Specify the signal at the slicer input and indicate the components which constitute 1.) the desired signal, 2.) ISI, and 3.) additive noise (still assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).

Problem 3 (20 credits)

Let us examine an orthogonal space-time code (a so-called “Alamouti code”) for two transmit antennas and one receive antenna:



Consider two time instants (t=0, t=T_s):

- Antenna 1 transmits the symbols A_1 $(t = 0)$ and $-A_2^*$ $(t = T_s)$.
- Antenna 2 transmits the symbols A_2 $(t = 0)$ and A_1^* $(t = T_s)$.

The transmit symbols $A_1, A_2 \in \{1, j, -1, -j\}$ are equally likely and statistically independent. H_1 and H_2 are the channel fading coefficients between the antennas. Y_i denotes the received signal and N_i the additive noise at time instant i. All variables are complex-valued. The noise is circularly symmetric Gaussian distributed with mean $\mathrm{E}\{\mathbf{n}\} = \mathbf{0}$ and correlation matrix $\mathbf{R_n} = \mathrm{E}\{\mathbf{n}\,\mathbf{n}^H\} = \sigma_n^2\,\mathbf{I}$, where $\mathbf{n} = (N_0\ N_1)^T$.

a) What is the bit rate R_b of the transmission? ($T_s = 0.7$ms)

b) The input-output relation of the transmission system can be expressed as follows (see figure above):

$$\underbrace{\begin{pmatrix} Y_0 \\ Y_1^* \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}}_{\mathbf{H}} \underbrace{\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}}_{\mathbf{a}} + \underbrace{\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}}_{\mathbf{w}} . \tag{1}$$

Specify $\mathbf{H}$ and $\mathbf{w}$.

c) Calculate the receive vector after the “matched filter”

$$\mathbf{z} = \mathbf{H}^H\,\mathbf{y} \tag{2}$$

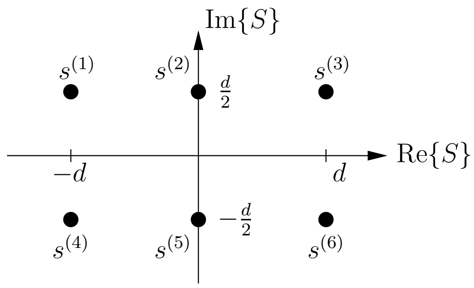
as a function of $\mathbf{a}$ and $\mathbf{w}$. Simplify $\mathbf{H}^H\mathbf{H}$ as far as possible.

d) Determine the mean and variance of the noise after the matched filter. Find the correlation matrix of the filtered noise. Are the two noise components still statistically independent after filtering?

e) Design an ML receiver for A_1 and A_2.

Problem 4 (20 credits)

A single symbol $A \in \{a^{(1)}, \dots, a^{(4)}\}$ is to be transmitted over a passband channel. At the transmitter, the symbol A is mapped to a symbol $S \in \{s^{(1)}, \dots, s^{(6)}\}$ from the following constellation:



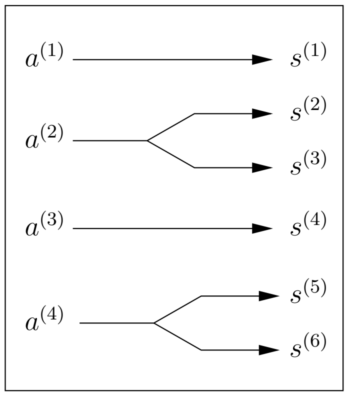
The mapping $A \to S$ is partly random according to the following relation:

$\mathrm{P}\{A = a^{(1)}\} = 1/8$

$\mathrm{P}\{A = a^{(2)}\} = 1/8$

$\mathrm{P}\{A = a^{(3)}\} = 1/4$

$\mathrm{P}\{A = a^{(4)}\} = 1/2$



$\mathrm{P}\{S = s^{(2)} | A = a^{(2)}\} = 3/4$

$\mathrm{P}\{S = s^{(3)} | A = a^{(2)}\} = 1/4$

$\mathrm{P}\{S = s^{(5)} | A = a^{(4)}\} = 3/4$

$\mathrm{P}\{S = s^{(6)} | A = a^{(4)}\} = 1/4$

The received value Q is given by $Q = S + Z$, where Z is circularly symmetric complex Gaussian noise with variance $\sigma_Z^2 = 0.09$.

a) Calculate the probabilities of the symbols $s^{(i)}$.

b) Calculate the symbol distance d such that the mean symbol power equals $\mathrm{E}\{|S|^2\} = 1$.

c) Sketch the ML decision regions for detecting S.

d) For the case $S = s^{(1)}$, calculate the exact error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_S | S = s^{(1)}\}$ of the ML detector for S. What is the error probability for A in this case?