Modulations- und Detektionsverfahren

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A random variable S from the symbol alphabet $\mathcal{S} = \{2, 2j, -2, -2j\}$ is transmitted to two different receivers through two different channels. One channel introduces additive noise, i.e., the received random variable is

$$Y_1 = S + N_1.$$

The other channel introduces multiplicative noise, i.e., the received random variable is

$$Y_2 = SN_2.$$

Both N_1 and N_2 are circularly symmetric complex Gaussian with mean 1 and variance 1. N_1 and N_2 are statistically independent of S.

a) Calculate $f_{Y_1|S}(y_1|s)$ for each $s \in \mathcal{S}$. Sketch the conditional mean of Y_1 for each $s \in \mathcal{S}$ and the decision regions of the ML receiver in a single diagram.

b) Same as item a), but for $f_{Y_2|S}(y_2|s)$.

c) Suppose that both receivers use the respective ML decision rule. Is the error probability of the second receiver larger than, equal to, or smaller than that of the first receiver? Explain why.

Problem 2 (20 credits)

Consider the following pulse set:

$$\begin{aligned} g_1(t) &= \operatorname{sinc}\left(\pi \frac{t}{T_s}\right) \cos\left(3\pi \frac{t}{T_s}\right) \\ g_2(t) &= \operatorname{sinc}\left(\frac{\pi}{2} \frac{t}{T_s}\right) \cos\left(\frac{3\pi}{2} \frac{t}{T_s}\right) \\ g_3(t) &= \operatorname{sinc}\left(\frac{\pi}{2} \frac{t}{T_s}\right) \cos\left(3\pi \frac{t}{T_s}\right). \end{aligned}$$

a) Which of the pulses satisfy the Nyquist criterion? (A derivation is required.)

b) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion.

c) Using the same pulses as in b), consider orthogonal multipulse PAM with symbol alphabet $\mathcal{A} = \{-d/2, d/2\}$. The symbols $A_m[k]$ are uniformly distributed and uncorrelated, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = \sigma_A^2 \delta_{k,l}\delta_{m,n}$. Calculate and sketch the power spectral density $S_{\bar{S}}(j\omega)$ of the stationarized transmit signal $\bar{S}(t)$.

Problem 3 (20 credits)

Consider a PAM system with received pulse

$$h(t) = e^{-t/T_s}\,\mathrm{u}(t) + e^{-(t/T_s - a)}\,\mathrm{u}(t - aT_s), \quad \text{where } a > 0.$$

Here, T_s is the symbol rate and $\mathrm{u}(t)$ denotes the unit step function. The noise $N(t)$ is white with power spectral density $N_0/2$. The receiver consists of a bandpass-lowpass transformation, a receive filter $f(t)$, a symbol-rate sampler, a linear ZF equalizer, and a slicer.

a) Specify the receive filter $f(t)$ that minimizes the noise enhancement of the ZF equalizer. Use this receive filter in the following.

b) Find a function $\phi(t)$ such that $h(t) = \phi(t) + \phi(t - aT_s)$. Calculate $r_\phi(\tau) = \int_{-\infty}^{\infty} \phi(t)\phi^*(t-\tau)\mathrm{d}t$.

c) Calculate the equivalent discrete-time pulse $p[k]$ at the equalizer input. *Hint:* Use your result from b).

d) Find the value of a such that $p[0] = \frac{3}{2}\,T_s$.

Problem 4 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{1+j, 1-j, -1+j, -1-j\}$. The symbols $A[k]$ are assumed white and uniformly distributed.

a) For transmission, the symbol sequence $A[k]$ is transformed into a sequence $B[k] = A[k] + \alpha A[k-1]$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transformed symbols $B[k]$.

b) The equivalent baseband transmit signal is $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

c) The spectrum $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is required to be zero at frequency $\omega = \frac{\pi}{T_s}$. Which choice of α satisfies this condition?

d) Assume that the transmit pulse $g(t)$ is a sinc pulse: $g(t) = \mathrm{sinc}(\frac{\pi t}{T_s})$. Sketch the power spectral density of the stationarized transmit signal $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ for this case, using the α calculated in c).