Digital Communications 1
Written exam on June 23, 2020

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A stationary and white bit sequence (bit rate $R_b = 64\,\text{kbit/s}$) is to be transmitted over a bandlimited AWGN channel (power spectral density $S_N(j\omega) = N_0/2 = 9{\cdot}10^{-7}\,\text{W/Hz}$, bandwidth $B_\text{T} = 24\,\text{kHz}$). The modulation scheme is passband PAM using an M_a-ary QAM constellation. The Fourier transform of the transmit pulse is given by $G(j\omega) = \sqrt{R(j\omega)}$, where $R(j\omega)$ is the Fourier transform of a raised-cosine pulse with roll-off factor $\alpha = 0.5$.

a) Determine the size of the symbol alphabet M_a and sketch the corresponding symbol constellation.

b) Design a receive filter for ISI-free transmission.

c) Sketch the block diagram of the ML receiver.

For the following, assume that symbols with positive real part are twice as likely than symbols with negative real part.

d) Calculate the minimum symbol distance d_a for a symbol error probability of $\text{P}\{\mathcal{E}_s\} = 10^{-7}$. Use for this calculation the approximation $\text{P}\{\mathcal{E}_s\} \approx \overline{\mathcal{N}}\mathcal{Q}\Big(\frac{d_\text{min}}{\sqrt{2}\sigma_Z}\Big)$.

e) How does the symbol constellation need to be shifted to minimize the mean transmit power $P_{\bar{S}}$? Sketch the shifted constellation. How large is the achieved reduction of $P_{\bar{S}}$?

Problem 2 (20 credits)

Consider a passband PAM system in which the Fourier transform of the total pulse $p(t)$ is given by

$$P(j\omega) = \mathrm{rect}\left(\omega; \frac{3\pi}{2T_s}\right).$$

Due to a sampling phase offset at the receiver, the sampling instants are $t = kT_s + \tau$.

a) Calculate the Fourier transform of the resulting equivalent discrete-time pulse, $P(e^{j\theta})$, for $\tau = T_s/2$. Provide separate expressions of $P(e^{j\theta})$ for the intervals $\theta \in [-\pi/2, \pi/2]$, $\theta \in [\pi/2, \pi]$, and $\theta \in [-\pi, -\pi/2]$ and simplify these expressions as much as possible. Sketch the real part of $P(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Repeat Part a) for $\tau = T_s/4$.

c) For $\tau = T_s/4$, calculate and sketch the transfer function of the ZF linear equalizer.

d) Assume sampling at double symbol rate, i.e., the sampling period is now $T_s' = T_s/2$. Furthermore assume that the sampling offset is $\tau = T_s'/2$. Calculate the resulting equivalent discrete-time pulse $P'(e^{j\theta})$.

Problem 3 (20 credits)

A binary random variable $S \in \{-1, 3/2\}$ with $p_S(-1) = 1/4$ is corrupted by additive noise N that is statistically independent of S and has a shifted Laplacian distribution

$$f_N(n) = ae^{-\lambda|n-1|}.$$

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

c) Calculate the parameter λ such that $\mathrm{P}\{N < 0\} = 1/7$. (This value of λ is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.

Problem 4 (20 credits)

Consider an orthogonal multipulse PAM system with M transmit pulses $g_m(t)$ and random transmit symbols $A_m[k]$, $m \in \{1, \ldots, M\}$, $k \in \mathbb{Z}$.

a) Derive an expression of the autocorrelation function $R_{\bar{S}}(\tau)$ of the stationarized transmit signal $\bar{S}(t)$ without making any assumptions about the symbol statistics.

b) Simplify $R_{\bar{S}}(\tau)$ from part a) for the case that the symbols $A_m[k]$ are zero-mean and uncorrelated with respect to m, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = R_A^{(m)}[k-l]\delta_{m,n}$. Calculate the power spectral density $S_{\bar{S}}(j\omega)$.

c) Further specialize $R_{\bar{S}}(\tau)$ and $S_{\bar{S}}(j\omega)$ from part b) for the case where the symbols $A_m[k]$ are moreover uncorrelated with respect to k and have constant (but still m-dependent) variances $\sigma^2_{A_m}$.

d) Assume transmit pulses $g_m(t) = G\cos\big((2m-1)\omega_0 t\big)\,\mathrm{sinc}\big(\pi\frac{t}{T_s}\big)$, $m \in \{1, \ldots, M\}$, where $\omega_0 = 2\pi/T_s$. Calculate and sketch $S_{\bar{S}}(j\omega)$ for zero-mean, uncorrelated, and uniformly distributed symbols $A_m[k] \in \{-2, 2\}$.

e) Determine the transmit bandwidth and spectral efficiency if the transmit pulses from part d) are used.