Modulations- und Detektionsverfahren

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider the following pulse set:

$$\begin{aligned}
g_1(t) &= \operatorname{sinc}\left(\pi \frac{t}{T_s}\right) \cos\left(3\pi \frac{t}{T_s}\right) \\
g_2(t) &= \operatorname{sinc}\left(\frac{\pi}{2} \frac{t}{T_s}\right) \cos\left(\frac{3\pi}{2} \frac{t}{T_s}\right) \\
g_3(t) &= \operatorname{sinc}\left(\frac{\pi}{2} \frac{t}{T_s}\right) \cos\left(3\pi \frac{t}{T_s}\right).
\end{aligned}$$

a) Which of the pulses satisfy the Nyquist criterion? (A derivation is required.)

b) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion.

c) Using the same pulses as in b), consider orthogonal multipulse PAM with symbol alphabet $\mathcal{A} = \{-1, 1\}$. The symbols $A_m[k]$ are uniformly distributed and uncorrelated, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = \delta_{k,l}\delta_{m,n}$. Sketch the power spectral density $S_{\bar{S}}(j\omega)$ of the stationarized transmit signal $\bar{S}(t)$.

Problem 2 (20 credits)

Consider transmission of the signals

$$s^{(i)}(t) = is(t), \quad i \in \{1, 2, 3\} \quad \text{with } p_I(1) = p_I(2)/2 = 1/4$$

over an AWGN channel with noise variance $N_0/2$. Here, $s(t)$ is a fixed signal with energy $E_s \neq 0$.

a) Determine an orthonormal basis of the transmit signal space.

b) Specify the vectors corresponding to the signals $s^{(i)}(t)$.

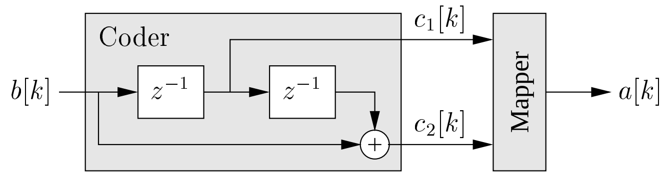
c) Sketch an implementation of the ML detector using inner products of the received signal with the basis functions.

d) Sketch an implementation of the ML detector using inner products of the received signal with the transmit signals.

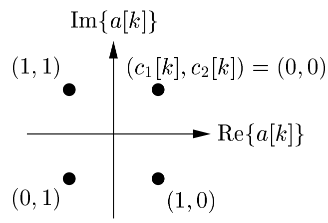
e) Calculate the conditional sequence error probabilities and the unconditional sequence error probability of the ML detector.

Problem 3 (20 credits)

Consider a system with trellis coded modulation (TCM). The transmitter consists of a convolutional encoder and a symbol mapper, as illustrated below.



TCM-Sender



Mapper

The convolutional encoder uses the data bits $b[k]$ to produce two code bits at each step,

$$c_1[k] = b[k-1] \quad \text{and} \quad c_2[k] = b[k] \oplus b[k-2]$$

($\oplus$ denotes addition modulo 2). The symbol mapper maps these code bits to symbols $a[k]$ (with $|a[k]|^2 = 2$).

The received signal is given by

$$y[k] = a[k] + w[k]$$

(ISI-free channel), with white complex Gaussian noise $w[k]$.

a) For the bit sequence $\{1, 1, 0, 1, 0\}$, derive the corresponding code bits $c_1[k]$, $c_2[k]$ and transmit symbols $a[k]$ (assuming that all previous data bits $b[k]$ are zero).

b) Make a list of all possible combinations of $b[k-2]$, $b[k-1]$, and $b[k]$ and the resulting $a[k]$.

c) Sketch a trellis diagram of the TCM transmitter.

d) The transmitted bit sequence is detected by an ML receiver that is implemented using the Viterbi algorithm. Calculate the detected bits $\hat{b}[k]$ from the received signal $y[0] = 3.1 - j1.4$, $y[1] = 0.2 + j1.2$, $y[2] = -0.7 + j0.9$ (assuming that $b[k] = 0$ for $k < 0$).

Problem 4 (20 credits)

Consider a PAM system with received pulse

$$h(t) = e^{-at}\,\mathrm{u}(t) + e^{-a(t-T_S)}\,\mathrm{u}(t-T_S), \quad \text{where } a > 0.$$

Here, T_S is the symbol rate and $\mathrm{u}(t)$ denotes the unit step function. The noise $N(t)$ is white with power spectral density $N_0/2$. The receiver consists of a bandpass-lowpass transformation, a receive filter $f(t)$, a symbol-rate sampler, a linear ZF equalizer, and a slicer.

- **a)** Specify the receive filter $f(t)$ that minimizes the noise enhancement of the ZF equalizer. Use this receive filter in the following.
- **b)** Find a function $\phi(t)$ such that $h(t) = \phi(t) + \phi(t - T_S)$. Use this relation for the following calculations.
- **c)** Calculate the equivalent discrete-time pulse $p[k]$ at the equalizer input.
- **d)** Find the value of a such that $p[1] = 2\,p[2]$.