Digital Communications 1

Written exam on May 8, 2014

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Asymmetric constellations provide a simple method for unequal error protection, where important bits can be protected more than bits of lesser importance. Consider an asymmetric QPSK constellation as shown in Figure 1, where the mapping of two bits b_1 b_2 into each signal point is also shown. The information bits are equally likely. Communication is performed over an AWGN channel with two-sided power spectral density $\frac{N_0}{2}$. The transmission is ISI-free with gain factor $|p[0]|$.

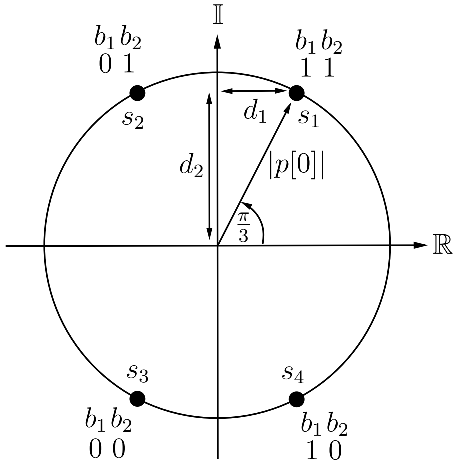

Figure 1: Asymmetric QPSK.

a) Draw the decision regions of the receiver that minimizes the symbol error probability.

b) Determine the symbol error probability of the receiver in a) as a function of $\frac{|p[0]|}{\sqrt{N_0 E_f}}$.

c) Determine the bit error probabilities for bits b_1 and b_2 separately. Which bit is more protected and why?

d) Assume that $N_0 = 10^{-6}$ and that $E_f = 1$. How large does $|p[0]|$ need to be set to achieve $\mathrm{P}\{\mathcal{E}_{b_1}\} \leq 10^{-3}$? How large is $\mathrm{P}\{\mathcal{E}_{b_2}\}$ at that value of $|p[0]|$?

Problem 2 (20 credits)

For the transmission of a single symbol $a \in \{1, -1\}$, the transmitter uses the following signals:

$$s^{(i)}(t) = \cos\left(\frac{\pi(100 + a^{(i)})}{T} t\right) \operatorname{rect}\left(t; \frac{T}{2}\right), \qquad i = 1, 2,$$

where $a^{(1)} = 1, a^{(2)} = -1$. The selected signal is transmitted over an AWGN channel with noise power spectral density $N_0/2$, yielding the received signal

$$y(t) = s(t) + n(t).$$

a) Sketch an implementation of the ML receiver that calculates the inner products of the received signal with the two transmit signals,

$$c^{(i)} = \langle y, s^{(i)} \rangle, \quad i = 1, 2.$$

Describe the decision rule of this receiver.

b) Show that the transmit signals are orthogonal.
Hint: $\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$.

c) Calculate $c^{(i)}$ as a function of a and $n(t)$.

d) The inner product consists of a signal component and a noise component, i.e.,

$$c^{(i)} = c_A^{(i)} + c_N^{(i)}.$$

State the pdf of $c_N^{(i)}$.

e) Assume that $a = 1$ was transmitted. Calculate $c_A^{(1)}$ and $c_A^{(2)}$ for that case and state the pdf of $c^{(1)}$ and $c^{(2)}$.

f) Finally, assume that the transmission is corrupted by an interferer $w(t)$:

$$y(t) = s(t) + w(t) + n(t) \quad \text{with} \quad w(t) = \cos\left(\frac{100\pi}{T} t\right).$$

Still assuming $a = 1$, calculate $c_A^{(1)}$ and $c_A^{(2)}$ and state the pdf of $c^{(1)}$ and $c^{(2)}$.

Problem 3 (20 credits)

A random variable S from the symbol alphabet $\mathcal{S} = \{1, j, -1, -j\}$ is transmitted to two different receivers through two different channels. One channel introduces additive noise, i.e., the received random variable is

$$Y_1 = S + N_1.$$

The other channel introduces multiplicative noise, i.e., the received random variable is

$$Y_2 = SN_2.$$

Both N_1 and N_2 are circularly symmetric complex Gaussian with mean $1/2$ and variance 1. Furthermore, N_1 and N_2 are statistically independent of S.

a) Calculate $f_{Y_1|S}(y_1|s)$ for each $s \in \mathcal{S}$. Sketch the conditional mean of Y_1 for each $s \in \mathcal{S}$ and the decision regions of the ML receiver in a single diagram.

b) Same as item a), but for $f_{Y_2|S}(y_2|s)$.

c) Suppose that both receivers use the respective ML decision rule. Is the error probability of the second receiver larger than, equal to, or smaller than that of the first receiver? Explain why.

Problem 4 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{7z^2 + 50z + 7}{7z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Find the poles and zeros of $S_h(z)$.

b) Find a minimum phase factorization of $S_h(z)$. $\lim_{n\to\infty} \frac{1}{n} \sum_{n'=1}^{n} \nu_n$

c) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

d) Consider the transmission of statistically independent symbols $A[k] \in \{1, j, -1, -j\}$ with equal probabilities. Assume that the Viterbi algorithm is used at the output of the noise whitening filter. Sketch the state transition diagram.

e) Assume that the receive filter is the matched filter, i.e., $f(t) = h^*(-t)$. Calulate the frequency response $D_{\mathrm{ZF}}(e^{j\theta})$ of the corresponding zero forcing equalizer.