Modulations- und Detektionsverfahren

Written exam on October 16, 2012

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A random variable A is chosen from alphabet $\mathcal{A}$ with equal probabilities. It is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of A and Gaussian distributed with mean $\mu_N = 0$ variance $\sigma_N^2 = 1/16$.

a) Consider $\mathcal{A} = \{-1, 1\}$. What is the mean power P_A? Find the ML decision rule and calculate the error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ of the ML detector.

b) Consider $\mathcal{A} = \{-1, 1, \beta\}$ with $\beta > 1$.

b1) Find the ML decision rule and calculate the conditional error probabilities $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}|A = a\}$ for $a \in \{-1, 1, \beta\}$ and the unconditional error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$.

b2) Find the value of β for which $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ is the same as in a). This value is to be used in what follows.

c) The alphabet from b) is shifted by a real constant δ, i.e., $\mathcal{A} = \{-1+\delta, 1+\delta, \beta+\delta\}$. Find the value of δ for which the mean power P_A is minimized. This value is to be used in what follows.

d) The alphabet from c) is scaled by a real constant γ, i.e., $\mathcal{A} = \{\gamma(-1+\delta), \gamma(1+\delta), \gamma(\beta+\delta)\}$. Find the value of γ for which the mean power P_A is the same as in a).

Problem 2 (20 credits)

Consider an AWGN channel with noise power spectrum $N_0/2 = 1.25 \cdot 10^{-5}$W/Hz and transmission of M_a-ary symbols using orthogonal multipulse modulation. The transmit pulses are assumed to satisfy the generalized Nyquist criterion; their energy is $E_g = 6,76 \cdot 10^{-4}$J. The symbols are detected by means of the ML sequence detector.

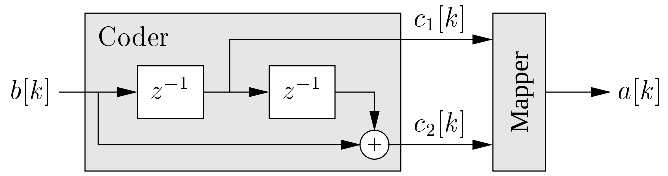
a) Using the nearest neighbor approximation, calculate the maximum number of symbols M_a such that a symbol error probability of less than $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\} = 1.55 \cdot 10^{-6}$ is achieved. This value of M_a is to be used in what follows.

b) Calculate the resulting symbol error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\}$ (again using the nearest neighbor approximation) and the bit error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_b\}$.

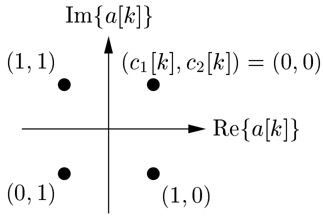
c) Calculate the maximum spectral efficiency ν_{max}, assuming bandpass transmission.

Problem 3 (20 credits)

Consider a system with trellis coded modulation (TCM). The transmitter consists of a convolutional encoder and a symbol mapper, as illustrated below.



TCM transmitter



Mapper

The convolutional encoder uses the data bits $b[k]$ to produce two code bits at each step,

$$c_1[k] = b[k-1] \quad \text{and} \quad c_2[k] = b[k] \oplus b[k-2]$$

($\oplus$ denotes addition modulo 2). The symbol mapper maps these code bits to symbols $a[k]$ (with $|a[k]|^2 = 2$).

The received signal is given by

$$y[k] = a[k] + w[k]$$

(ISI-free channel), with white complex Gaussian noise $w[k]$.

a) For the bit sequence $(1, 0, 1, 1, 0)$, derive the corresponding code bits $c_1[k]$, $c_2[k]$ and transmit symbols $a[k]$ (assuming that all previous data bits $b[k]$ are zero).

b) Provide a list of all possible combinations of $b[k–2]$, $b[k–1]$, and $b[k]$ and the resulting $a[k]$.

c) Sketch a trellis diagram of the TCM transmitter.

d) The transmitted bit sequence is detected by an ML receiver that is implemented using the Viterbi algorithm. Calculate the detected bits $\hat{b}[k]$ from the received signal $y[0] = -3.1 - j1.4$, $y[1] = -0.2 + j1.2$, $y[2] = -0.7 - j0.9$ (assuming that $b[k] = 0$ for $k < 0$).

Problem 4 (20 credits)

In a passband PAM system, the received pulse is given by

$$h(t) = \frac{1}{\sqrt{T_s}} \operatorname{rect}\left(t; \frac{T_s}{3}\right) - \frac{1}{2\sqrt{T_s}} \operatorname{rect}\left(t - \frac{2}{3}T_s; \frac{T_s}{3}\right).$$

a) Calculate the impulse response $\rho_h[k]$ and the transfer function $S_h(z)$ of the equivalent dicrete-time system (including the sampled matched filter).

b) Find the zeros and poles of $S_h(z)$.

c) Calculate the linear *zero forcing* equalizer and find its poles.

d) Find a minimum-phase factorization of $S_h(z)$.

e) Calculate the transfer function and the impulse response of the equivalent discrete-time system including the *noise-whitening* filter.