Digital Communications 1
Written exam on December 6, 2018

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-1, 1, -2j\}$. All three symbols are equally likely. The symbols $A[k]$ are assumed statistically independent.

a) Calculate the symbol power P_A. How should the symbol alphabet be shifted in order to minimize the symbol power?

In the following, use the shifted symbol alphabet.

b) Prior to transmission, the symbol sequence $A[k]$ is transformed into the sequence $B[k] = A[k] + \alpha A[k-1]$ with $\alpha \in \mathbb{R}$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transmitted symbols $B[k]$.

c) The transmit signal in the equivalent baseband domain is given by $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

d) Determine α such that $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is zero at frequency $\omega = \frac{\pi}{T_s}$.

Problem 2 (20 credits)

Consider equalization of a channel with equivalent discrete-time baseband pulse

$$p[k] = \delta[k] + \delta[k-1] + \delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1$.

a) Determine $D_{\mathrm{ZF}}(e^{j\theta})$.

b) Now, assume that an MMSE equalizer with impulse response $d_{\mathrm{MSE}}[k], k \in [-L, L]$ is used. What is a suitable value for L? Why?

c) Sketch the block diagram of an MMSE equalizer using L as determined in b).

d) Determine $d_{\mathrm{MSE}}[k]$ for $k \in [-L, L]$.

Hint: The inverse of a matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

can be calculated as

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{13}a_{32} - a_{12}a_{33} & a_{12}a_{23} - a_{13}a_{22} \\ a_{23}a_{31} - a_{21}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{13}a_{21} - a_{11}a_{23} \\ a_{21}a_{32} - a_{22}a_{31} & a_{12}a_{31} - a_{11}a_{32} & a_{11}a_{22} - a_{12}a_{21} \end{pmatrix},$$

with

$$\det \mathbf{A} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

Problem 3 (20 credits)

A binary random variable $S \in \{-2, 2\}$ with $p_S(2) = 1/4$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & \text{if } n \geq b, \\ 0 & \text{if } n < b, \end{cases}$$

with a given $b \in \mathbb{R}^+$.

a) Calculate the probabilities $\mathrm{P}\{N < n_0\}$ and $\mathrm{P}\{N > n_0\}$ for $n_0 \in \mathbb{R}$.

b) Calculate the parameter $b \in \mathbb{R}^+$ such that $\mathrm{P}\{N > 2\} = 1/2$. (This value of b is to be used in what follows.)

Consider detection of S from $Y = S + N$.

c) Determine and sketch the ML decision rule.

d) Determine and sketch the MAP decision rule.

Problem 4 (20 credits)

A sequence of symbols $a[k] \in \{-1, 0, 1\}$, with $a[k] = 0$ for $k < 0$ is transmitted over a discrete-time channel with impulse response $h[k] = \delta[k] - 0.5\,\delta[k-1]$. The additive noise is white and Gaussian. The received sequence $y[k]$ is given by $y[0] = 0.8$, $y[1] = 0.2$, and $y[2] = -1.1$.

a) Represent this channel by a shift register circuit, a state diagram, and an elementary stage of the corresponding trellis diagram.

b) Use the Viterbi algorithm for ML sequence detection. Which sequence $\hat{a}[k]$ ($k = 0, 1, 2$) is obtained?

c) An alternative receiver uses a zero-forcing equalizer followed by a slicer. Which sequence $\hat{a}[k]$ ($k = 0, 1, 2$) is obtained with this receiver? You may assume that $y[k] = 0$ for $k < 0$.