Digital Communications 1
Written exam on December 5, 2016

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-1, 1, -j, j\}$. The symbols $A[k]$ are assumed white and uniformly distributed.

a) For transmission, the symbol sequence $A[k]$ is transformed into a sequence $B[k] = A[k] + \alpha A[k-1]$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transformed symbols $B[k]$.

b) The equivalent baseband transmit signal is $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t-kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

c) The spectrum $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is required to be zero at frequency $\omega = \frac{\pi}{T_s}$. Which choice of α satisfies this condition?

d) Assume that the transmit pulse $g(t)$ is a sinc pulse: $g(t) = \mathrm{sinc}(\frac{\pi t}{T_s})$. Sketch the power spectral density of the stationarized transmit signal $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ for this case, using the α calculated in c).

Problem 2 (20 credits)

Consider a passband PAM system with symbol period T_s, transmit pulse $g(t)$ with energy $E_g = 1$, and transmit symbols $A[k]$. The receive pulse in the equivalent baseband is $h(t) = g(t) + \alpha g(t - T_s)$ with $|\alpha| < 1$. The additive channel noise has a power spectral density of $N_0/2$. The receiver consists of a sampled matched filter, a noise whitening filter (based on a *causal* factorization), a slicer, and an equalizer.

a) Determine the impulse response, transfer function, and noise power spectral density of the equivalent discrete-time model of the transmission system including the noise whitening filter but not the equalizer.

b) Calculate the zero-forcing equalizer (placed after the noise whitening filter and slicer). Compare the noise variance at the input and output of the equalizer.

c) Verify that the output of the noise whitening filter is free of precursor intersymbol interference.

d) Determine the decision-feedback equalizer that completely removes postcursor intersymbol interference. (The noise whitening filter is used as the feedforward filter of the decision-feedback equalizer.)

Problem 3 (20 credits)

A random variable A is chosen from alphabet $\mathcal{A} \subseteq \mathbb{R}$ with equal probabilities. It is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of A and Gaussian with mean $\mu_N = 0$ variance $\sigma_N^2 = 1/16$.

a) Consider $\mathcal{A} = \{-1, 1\}$. What is the mean power P_A? Find the ML decision rule and calculate the error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ of the ML detector.

b) Consider $\mathcal{A} = \{-1, 1, \beta\}$ with $\beta > 1$.

- **b1)** Find the ML decision rule and calculate the conditional error probabilities $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}|A = a\}$ for $a \in \{-1, 1, \beta\}$ and the unconditional error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$.
- **b2)** Find the value of β for which $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ is the same as in a). This value is to be used in what follows.

c) The alphabet from b) is shifted by a real constant δ, i.e., $\mathcal{A} = \{-1 + \delta, 1 + \delta, \beta + \delta\}$. Find the value of δ for which the mean power P_A is minimized. This value is to be used in what follows.

d) The alphabet from c) is scaled by a real constant γ, i.e., $\mathcal{A} = \{\gamma(-1+\delta), \gamma(1+\delta), \gamma(\beta+\delta)\}$. Find the value of γ for which the mean power P_A is the same as in a).

Problem 4 (20 credits)

Consider an AWGN channel with noise power spectrum $N_0/2 = 1.25 \cdot 10^{-5}$W/Hz and transmission of M_a-ary symbols using orthogonal multipulse modulation. The transmit pulses are assumed to satisfy the generalized Nyquist criterion; their energy is $E_g = 6.76 \cdot 10^{-4}$J. The symbols are detected by the ML sequence detector.

a) Using the "nearest neighbor approximation" (i.e., the approximation of the symbol error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\}$ by the union bound), calculate the maximum number of symbols M_a such that a symbol error probability of less than $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\} = 1.55 \cdot 10^{-6}$ is achieved. This value of M_a is to be used in what follows.

b) Calculate the resulting symbol error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\}$ (again using the nearest neighbor approximation) and the bit error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_b\}$.

c) Calculate the maximum spectral efficiency $\nu_{\max}$, assuming bandpass transmission.