Digital Communications 1
Written exam on December 10, 2015

Institute of Telecommunications

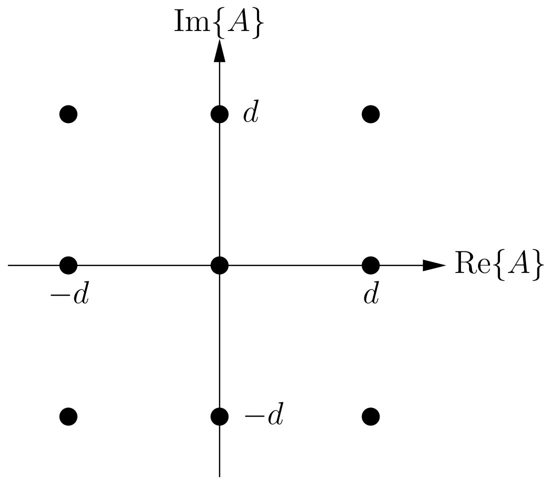
Vienna University of Technology

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider the transmission of a single symbol A from the following signal constellation:



The transmit probabilities are

$$P[A = \pm d] = P[A = \pm jd] = 1/8 \quad \text{and} \quad P[A = (\pm 1 \pm j)d] = 1/16.$$

The received symbol is $Q = A + Z$, where Z is circularly symmetric complex Gaussian noise with variance $\sigma_Z^2 = d^2/5$. Symbol A and noise Z are statistically independent.

a) Calculate the symbol mean μ_A and the symbol variance σ_A^2.

b) Calculate and sketch the decision regions of the MAP receiver.

c) Calculate the error probability of the MAP receiver.

d) Calculate the error probability of the ML receiver.

Problem 2 (20 credits)

Consider a bandpass PAM system with symbol alphabet $\mathcal{A} = \{-2, 2, -4j\}$. All three symbols are equally likely. The symbols $A[k]$ are assumed statistically independent.

a) Calculate the symbol power P_A. How should the symbol alphabet be shifted in order to minimize the symbol power?

In the following, use the shifted symbol alphabet.

b) Prior to transmission, the symbol sequence $A[k]$ is transformed into the sequence $B[k] = A[k] - \alpha A[k-2]$ with $\alpha \in \mathbb{R}$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transmitted symbols $B[k]$.

c) The transmit signal in the equivalent baseband domain is given by $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t - kT_s)$, where $g(t)$ is the Nyquist pulse with minimum bandwidth for a given T_s. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

d) Determine α such that $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is zero at the frequency $\omega = \frac{\pi}{2T_s}$.

Problem 3 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{10z^2 + 101z + 10}{10z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of symbols $A[k] \in \{1, j, -1, -j\}$. Sketch the state transition diagram associated with the equivalent discrete-time system including the noise whitening filter.

Problem 4 (20 credits)

A binary random variable $S \in \{-1, 3/2\}$ (with $p_S(-1) = 1/4$) is corrupted by additive noise N that is statistically independent of S and has a shifted Laplacian distribution

$$f_N(n) = a\, e^{-\lambda|n-1|}.$$

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

c) Calculate the parameter λ such that $\mathrm{P}\{N < 0\} = 1/7$. (This value of λ is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.