Modulations- und Detektionsverfahren

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider passband PAM transmission of statistically independent symbols $a[k] \in \{1, 0, -1\}$ with equal probabilities over an AWGN channel. The receiver uses the Viterbi algorithm to implement the ML sequence detector. The impulse response of the equivalent discrete-time system including the noise whitening filter is

$$c_h[k] = 6\,\delta[k] - 5\,\delta[k-1] + 2\,\delta[k-2]\,.$$

a) Assume that a sequence of four symbols $a[1], a[2], a[3], a[4]$ is to be detected. How many possible sequences are there?

b) Let $b[k]$ denote the output of the filter (with impulse response $c_h[k]$) within the equivalent discrete-time system. Complete the tables below with the missing values of $b[k]$:

$a[k-2]$	$a[k-1]$	$a[k]$	$b[k]$
1	1	1	3
1	1	0	−3
1	1	−1	−9
1	0	1	8
1	0	0	2
1	0	−1	−4
1	−1	1	
1	−1	0	
1	−1	−1	1
0	1	1	1
0	1	0	−5
0	1	−1	−11
0	0	1	6
0	0	0	0

$a[k-2]$	$a[k-1]$	$a[k]$	$b[k]$
0	0	−1	−6
0	−1	1	11
0	−1	0	5
0	−1	−1	
−1	1	1	
−1	1	0	−7
−1	1	−1	−13
−1	0	1	4
−1	0	0	−2
−1	0	−1	−8
−1	−1	1	9
−1	−1	0	3
−1	−1	−1	−3

c) How many symbols are contained in the state Ψ_k? How many possible states Ψ_k are there? Sketch one stage of the trellis diagram.

d) At the output of the equivalent discrete-time system, the following sequence is observed: $w[1] = -3$, $w[2] = 0$, $w[3] = -8$, $w[4] = 4$. Sketch a trellis diagram with Ψ_1, Ψ_2, Ψ_3, Ψ_4, and Ψ_5 for the case that $a[1] = 1$ and $a[2] = -1$ are known. Find the optimal path in the trellis diagram and specify $\hat{a}[3]$, $\hat{a}[4]$, and $\hat{\Psi}_1$.

Problem 2 (20 credits)

Consider a passband PAM system in which the Fourier transform of the total pulse $p(t)$ is given by

$$P(j\omega) = \mathrm{rect}\left(\omega; \frac{3\pi}{2T_s}\right).$$

Due to a sampling phase offset at the receiver, the sampling instants are $t = kT_s + \tau$.

a) Calculate the Fourier transform of the resulting equivalent discrete-time pulse, $P(e^{j\theta})$, for $\tau = T_s/2$. Provide separate expressions of $P(e^{j\theta})$ for the intervals $\theta \in [-\pi/2; \pi/2]$, $\theta \in [\pi/2; \pi]$, and $\theta \in [-\pi; -\pi/2]$ and simplify these expressions as much as possible. Sketch the real part of $P(e^{j\theta})$ for $\theta \in [-\pi; \pi]$.

b) Repeat part a) for $\tau = T_s/4$.

c) For $\tau = T_s/4$, calculate and sketch the transfer function of the ZF linear equalizer.

d) Assume sampling at double symbol rate, i.e., the sampling period is now $T_s' = T_s/2$. Furthermore assume that the sampling offset is $\tau = T_s'/2$. Calculate the resulting equivalent discrete-time pulse, $P'(e^{j\theta})$.

Problem 3 (20 credits)

Consider the following pulse set:

$$\begin{aligned}
g_1(t) &= \operatorname{sinc}\left(\frac{\pi}{4}\frac{t}{T_s}\right)\cos\left(\frac{3\pi}{4}\frac{t}{T_s}\right)\\
g_2(t) &= g_1(2t)\\
g_3(t) &= \operatorname{sinc}\left(\frac{\pi}{2}\frac{t}{T_s}\right)\cos\left(2\pi\frac{t}{T_s}\right)\\
g_4(t) &= g_2(2t).
\end{aligned}$$

a) Sketch the pulses in the frequency domain.

b) Which of the pulses satisfy the Nyquist criterion? (A derivation is required.)

c) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion.

Problem 4 (20 credits)

A random variable $S \in \{s^{(i)}\}_{i=1}^{4}$ with

$$s^{(1)} = -3, \quad s^{(2)} = -1, \quad s^{(3)} = 1, \quad s^{(4)} = 3$$

is distributed according to

$$p_I(1) = \frac{1}{3}, \quad p_I(2) = \frac{1}{6}, \quad p_I(3) = \frac{1}{6}, \quad p_I(4) = \frac{1}{3}\,.$$

S is corrupted by additive noise N, yielding the observation

$$Y = S + N\,.$$

Here, N is statistically independent of S and distributed according to the Cauchy distribution:

$$f_N(n) = \frac{1/\pi}{n^2 + 1}\,.$$

a) Calculate the probabilities $P\{N > n_1\}$ and $P\{N < n_2\}$. *Hint*: $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$.

b) Calculate the likelihood function $f_{Y|I}(y|i)$ and the posterior distribution $p_{I|Y}(i|y)$.

c) Calculate and sketch the MAP decision rule $\hat{\imath}_{\mathrm{MAP}}(y)$.

d) Calculate the conditional error probability of the MAP detector for each $i \in \{1, \ldots, 4\}$.

e) Calculate the unconditional error probability of the MAP detector.