Modulations- und Detektionsverfahren

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A binary random variable $S \in \{1, -1\}$ with prior probabilities p_1 and $1-p_1$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and distributed as follows:

$$f_N(n) = \alpha\left[(2+n)\,\mathrm{rect}(1+n;1) + (2-2n)\,\mathrm{rect}\Big(-\frac{1}{2}+n;\frac{1}{2}\Big)\right], \quad n \in \mathbb{R}\,.$$

a) Sketch $f_N(n)$ and calculate α.

b) Sketch $f_{Y|S}(y|s)$ for $s \in \{-1, 1\}$. Calculate the ML decision threshold η_{ML}.

c) Calculate the conditional error probabilities $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}|S = s\}$ for $s \in \{-1, 1\}$ and the unconditional error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}\}$ of the ML detector.

d) Calculate the MAP decision threshold η_{MAP} as a function of p_1.

e) Calculate the value of p_1 for which

e1) $\eta_{\mathrm{MAP}} = \eta_{\mathrm{ML}}$;

e2) $\eta_{\mathrm{MAP}} = 0$;

e3) $\eta_{\mathrm{MAP}} = -\frac{2}{3}$.

Problem 2 (20 credits)

Consider passband PAM transmission of symbols $A[k]$ that are taken from the alphabet $\{2, -2\}$ with equal probabilities. The symbol sequence $A[k]$ and the equivalent discrete-time noise $Z[k]$ are independent and both white. The noise is zero-mean and complex Gaussian with variance $\sigma_Z^2 = 1/4$. The receiver consists of a receive filter and a slicer with decision threshold 0. The equivalent discrete-time baseband pulse is

$$p[k] = \delta[k] - \frac{1}{2}\delta[k-1].$$

a) Specify the signal $Q[k]$ at the slicer input and indicate the components constituting 1) the desired signal, 2) ISI, and 3) additive noise.

b) Find the likelihood function $f_{Q[k]|A[k],A[k-1]}\big(q[k]\big|a[k], a[k-1]\big)$ and calculate the conditional error probabilities $\mathrm{P}\big\{\hat{A}[k] \neq A[k]\big|A[k], A[k-1]\big\}$ for all four cases $(A[k], A[k-1]) \in \{2, -2\}^2$. Calculate the unconditional error probability $\mathrm{P}\big\{\hat{A}[k] \neq A[k]\big\}$.

c) Calculate the mean powers of the three signal components from a).

Consider an extended receiver using a decision feedback equalizer that minimizes the mean-square error at the slicer input. The coefficients of the feedforward filter and of the feedback filter are $\mathbf{d}_{\mathrm{MSE}} = (8/293) \cdot (-1\ \ 34\ \ 0)^T$ and $\mathbf{v}_{\mathrm{MSE}} = (136/293) \cdot (-1\ \ 0)^T$, respectively.

d) Specify the signal at the slicer input and indicate the components constituting 1) the desired signal, 2) ISI, and 3) additive noise (assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).

Problem 3 (20 credits)

Consider passband PAM transmission over an AWGN channel. The receiver uses an ML sequence detector. The equivalent discrete-time baseband system including the sampled matched filter is described by the folded spectrum

$$S_h(z) = \frac{9z^2 - 82z + 9}{9z}.$$

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find the minimum-phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise-whitening filter.

e) Consider the transmission of statistically independent symbols $a[k] \in \{1, -1+j, -1-j\}$ with equal probabilities. Sketch the state transition diagram corresponding to the impulse response calculated in d).

Problem 4 (20 credits)

For two complex signals $x(t)$ and $y(t)$, consider the quantities $\|x\|$, $\|y\|$, and $\langle x, y\rangle$.

a) Show how these quantities are changed by the following transformations of *both* signals $x(t)$ and $y(t)$:

a1) a time shift by t_0;

a2) multiplication by a constant factor $\alpha \in \mathbb{C}$.

b) Consider the signals

$$x(t) = A\cos(\omega_0 t)\ \mathrm{rect}(t; T), \qquad y(t) = -A\cos(\omega_0 t + \theta)\ \mathrm{rect}(t - T; T),$$

with $T = 2\pi/\omega_0$.

b1) Calculate $\|x\|$, $\|y\|$, and $\langle x, y\rangle$.

b2) Sketch $\langle x, y\rangle$ as a function of θ.