Digital Communications 1
Written exam on October 19, 2017

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-1, 1, -2j\}$. All three symbols are equally likely. The symbols $A[k]$ are assumed statistically independent.

a) Calculate the symbol power P_A. How should the symbol alphabet be shifted in order to minimize the symbol power?

In the following, use the shifted symbol alphabet.

b) Prior to transmission, the symbol sequence $A[k]$ is transformed into the sequence $B[k] = A[k] + \alpha A[k-1]$ with $\alpha \in \mathbb{R}$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transmitted symbols $B[k]$.

c) The transmit signal in the equivalent baseband domain is given by $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k]g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

d) Determine α such that $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is zero at the frequency $\omega = \frac{\pi}{T_s}$.

Problem 2 (20 credits)

White and equally likely symbols $A[k] \in \{-1, 1\}$ are transmitted over a discrete-time channel with linear distortion described by the impulse response $p[k] = \delta[k] - \frac{1}{2}\delta[k-1]$ and additive white Gaussian noise $Z[k]$ of variance σ_Z^2. The noise $Z[k]$ is statistically independent of the symbol sequence $A[k]$. At the output of the channel, the sequence $Y[k] = (p * A)[k] + Z[k]$ is received.

a) As a first step, consider direct symbolwise detection of $Y[k]$.

a1) Calculate the conditional probability of a symbol error

$$\mathrm{P}\{\hat{A}[k] \neq A[k] \mid A[k] = \alpha, A[k-1] = \beta\} \qquad \text{for } \alpha, \beta \in \{-1, 1\}.$$

a2) Calculate the unconditional probability of a symbol error $\mathrm{P}\{\hat{A}[k] \neq A[k]\}$.

b) Now zero-forcing equalization of the sequence $Y[k]$ is considered prior to symbolwise detection.

b1) Find the transfer function and impulse response of the zero-forcing equalizer and calculate the noise variance after equalization.

b2) Calculate the unconditional probability of a symbol error $\mathrm{P}\{\hat{A}[k] \neq A[k]\}$ of this zero-forcing detector and compare it with the probability calculated in part a2).

Problem 3 (20 credits)

A binary random variable $S \in \{-2, 2\}$ with $p_S(2) = 1/4$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & \text{if } n \geq b, \\ 0 & \text{if } n < b, \end{cases}$$

with a given $b \in \mathbb{R}$.

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

c) Calculate the parameter $b \in \mathbb{R}$ such that $\mathrm{P}\{N > 2\} = 1/2$. (This value of b is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.

Problem 4 (20 credits)

A sequence of symbols $a[k] \in \{-1, 0, 1\}$, with $a[k] = 0$ for $k < 0$, is transmitted over a discrete-time channel with impulse response $h[k] = \delta[k] - 0.5\,\delta[k-1]$. The additive noise is white and Gaussian. The received sequence $y[k]$ is given by $y[0] = 0.8$, $y[1] = 0.2$, and $y[2] = -1.1$.

a) Represent this channel by a shift register circuit, a state diagram, and an elementary stage of the corresponding trellis diagram.

b) Use the Viterbi algorithm for ML sequence detection. Which sequence $\hat{a}[k]$ $(k = 0, 1, 2)$ is obtained with this receiver?

c) An alternative receiver uses a zero-forcing equalizer followed by a slicer. Which sequence $\hat{a}[k]$ $(k = 0, 1, 2)$ is obtained with this receiver? You may assume that $y[k] = 0$ for $k < 0$.