Digital Communications 1
Written exam on November 9, 2020

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider passband PAM transmission using the QPSK signal constellation $\mathcal{A} = \{1, j, -1, -j\}$. The transmission is ISI-free with complex gain factor $p[0] = 1 - j$. The receiver performs symbolwise ML detection. The real part and the imaginary part of the noise at the slicer input, $z[k] = z_{\mathrm{R}}[k] + j z_{\mathrm{I}}[k]$, are statistically independent zero-mean Gaussian random variables with variances $\sigma_{\mathrm{R}}^2 = 0.5$ and $\sigma_{\mathrm{I}}^2 = 1$, respectively.

a) Sketch the signal constellation.

b) Sketch the signal constellation as seen at the slicer input and the decision regions of the symbolwise ML detector.

c) Assume that the symbol $-j$ was transmitted. Calculate the probabilities of detecting 1, -1, and j. Calculate the conditional symbol error probability for the case that $-j$ was transmitted.

d) Calculate the conditional symbol error probabilities for the cases that 1, -1, and j were transmitted. Calculate the unconditional symbol error probability.

e) Assume that the transmitter decides to use only the symbols j and $-j$ but the receiver is not changed. How does this influence the unconditional symbol error probability?

Problem 2 (20 credits)

In a passband PAM system, the received pulse is given by

$$h(t) = \frac{1}{\sqrt{T_s}} \mathrm{rect}\left(t; \frac{T_s}{3}\right) - \frac{1}{2\sqrt{T_s}} \mathrm{rect}\left(t - \frac{2}{3}T_s; \frac{T_s}{3}\right).$$

a) Calculate the impulse response $\rho_h[k]$ and the transfer function $S_h(z)$ of the equivalent discrete-time system (including the sampled matched filter).

b) Find the zeros and poles of $S_h(z)$.

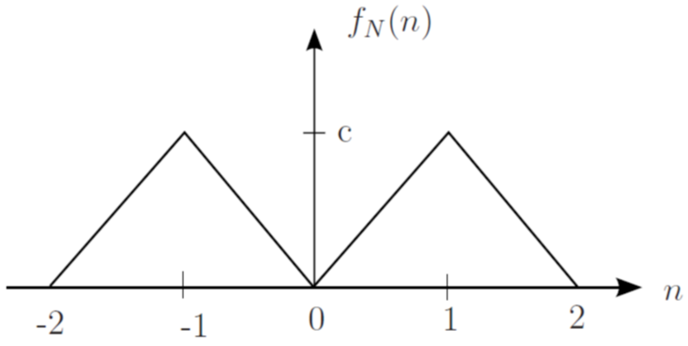
c) Calculate the linear zero forcing equalizer and find its poles.

d) Find a minimum-phase factorization of $S_h(z)$.

e) Calculate the transfer function and the impulse response of the equivalent discrete-time system including the noise-whitening filter.

Problem 3 (20 credits)

Consider transmission of a symbol $A \in \{-1, 0\}$ over a channel with additive noise N, which is distributed as shown below.



The transmission probabilities are defined as $\mathrm{P}\{A = 0\} = p$.

a) Calculate the constant c and the MAP decision rule.

b) Calculate the ML decision rule and the corresponding symbol error probability.

c) In the following assume that the receiver knows the sign of the noise N and uses this additional information for detection.

- **c1)** Assume $N > 0$. Sketch the conditional noise pdf for $N > 0$.
- **c2)** Calculate the ML decision rule for $N > 0$ and for $N < 0$.
- **c3)** How large is the symbol error probability of the ML detector that knows the sign of N?

Problem 4 (20 credits)

Consider passband PAM transmission of symbols $A[k]$ that are taken from the alphabet $\{2, -2\}$ with equal probabilities. The symbol sequence $A[k]$ and the equivalent discrete-time noise $Z[k]$ are independent and both white. The noise is zero-mean and complex Gaussian with variance $\sigma_Z^2 = 1/4$. The receiver consists of a receive filter and a slicer with decision threshold 0. The equivalent discrete-time baseband pulse is

$$p[k] = \delta[k] - \frac{1}{2}\delta[k-1].$$

a) Specify the signal $Q[k]$ at the slicer input and indicate the components constituting 1) the desired signal, 2) ISI, and 3) additive noise.

b) Find the likelihood function $f_{Q[k]|A[k],A[k-1]}(q[k]|a[k], a[k-1])$ and calculate the conditional error probabilities $\mathrm{P}\{\hat{A}[k] \neq A[k]|A[k], A[k-1]\}$ for all four cases $(A[k], A[k-1]) \in \{2, -2\}^2$. Calculate the unconditional error probability $\mathrm{P}\{\hat{A}[k] \neq A[k]\}$.

c) Calculate the mean powers of the three signal components from a).

Consider an extended receiver using a decision feedback equalizer that minimizes the mean-square error at the slicer input. The coefficients of the feedforward filter and of the feedback filter are $\mathbf{d}_{\mathrm{MSE}} = (-1, 34, 0)^T$ and $\mathbf{v}_{\mathrm{MSE}} = (-1, 0)^T$, respectively.

d) Specify the signal at the slicer input and indicate the components constituting 1) the desired signal, 2) ISI, and 3) additive noise (assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).