Digital Communications 1
Written exam on October 13, 2015

Institute of Telecommunications

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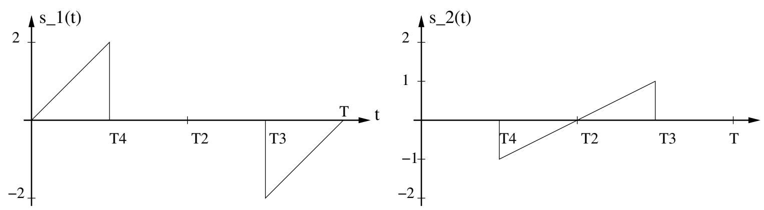
Please note:

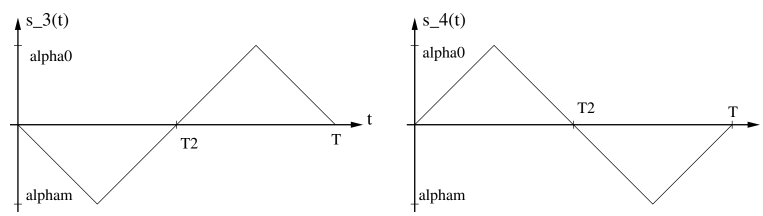
- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

For the transmission of a binary symbol (both values are equally likely) over a channel corrupted by additive white Gaussian noise with power spectral density $N_0/2$, two different signals are used. Let us compare the following signal sets:



SM1



SM2

a) Represent all four signals in terms of a common orthonormal basis that is as simple as possible. Specify that basis.

b) Calculate the amplitude $\alpha > 0$ for signal set II such that the average signal energy is equal for both signal sets.

c) For the detection of the transmitted binary symbol, an optimum receiver is employed. Calculate the error probabilities obtained with signal set I and signal set II using the amplitude α determined in Part b).

d) Calculate the amplitude $\alpha > 0$ such that the error probability obtained with signal set II is equal to the error probability obtained with signal set I.

Problem 2 (20 credits)

A transmission system uses the two transmit signals

$$s_1(t) = \sin(\omega_0 t)\ \mathrm{rect}(t - T; T)$$
$$s_2(t) = \sin(\omega_0 t + \theta)\ \mathrm{rect}(t - T; T),$$

with $T = \pi/\omega_0$ and $\omega_0 > 0$.

a) Calculate $\langle s_1, s_2 \rangle$ and sketch it as a function of θ.

b) Calculate $\|s_1\|$, $\|s_2\|$, and $\|s_1 - s_2\|$.

Consider transmission of $s_i(t)$, $i \in \{1, 2\}$ over an AWGN channel:

$$y(t) = s_i(t) + n(t),$$

where the noise $n(t)$ has a power spectral density of $N_0/2$.

c) Find the decision rule of the ML detector $\hat{i}_{\mathrm{ML}}(y)$. Sketch an implementation of the ML detector that uses only one inner product.

d) Using your results from Part b), calculate the error probability of the ML detector.

Problem 3 (20 credits)

Over a discrete-time channel with impulse response $h[k] = \delta[k] - 0.4 \cdot \delta[k-1]$, a sequence of symbols $a[k] \in \{-1, 0, 1\}$ is transmitted, with $a[k] = 0$ for $k < 0$. The additive noise is white and Gaussian. The received sequence $y[k]$ is given by $y[0] = 1.3$, $y[1] = 0.4$, and $y[2] = -1.2$.

a) Visualize this channel by a shift register circuit, a state diagram, and an elementary stage of the corresponding trellis diagram.

b) Use the Viterbi algorithm for ML sequence detection. Which sequence $\hat{a}[k]$ ($k = 0, 1, 2$) is obtained with this receiver?

c) An alternative receiver uses a zero-forcing equalizer followed by a slicer. Which sequence $\hat{a}[k]$ ($k = 0, 1, 2$) is obtained with this receiver? You may assume that $y[k] = 0$ for $k < 0$.

Problem 4 (20 credits)

Consider a passband PAM system in which the Fourier transform of the total pulse $p(t)$ is given by

$$P(j\omega) = \mathrm{rect}\left(\omega; \frac{7\pi}{4T_s}\right).$$

Due to a sampling phase offset at the receiver, the sampling instants are $t = kT_s + \tau$.

a) Calculate the Fourier transform of the resulting equivalent discrete-time pulse, $P(e^{j\theta})$, for $\tau = T_s/2$. Provide separate expressions of $P(e^{j\theta})$ for the intervals $\theta \in [-\pi/2; \pi/2]$, $\theta \in [\pi/2; \pi]$, and $\theta \in [-\pi; -\pi/2]$ and simplify these expressions as much as possible. Sketch the real part of $P(e^{j\theta})$ for $\theta \in [-\pi; \pi]$.

b) Repeat Part a) for $\tau = T_s/4$.

c) For $\tau = T_s/4$, calculate and sketch the transfer function of the ZF linear equalizer.

d) Assume sampling at double symbol rate, i.e., the sampling period is now $T_s' = T_s/2$. Furthermore assume that the sampling offset is $\tau = T_s'/2$. Calculate the resulting equivalent discrete-time pulse, $P'(e^{j\theta})$.