Modulations- und Detektionsverfahren

Written exam on November 17, 2010

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Binary symbols $a[k] \in \{1, -1\}$ are transmitted by means of passband PAM over a dispersive channel with additive noise. The transmit symbols are equally likely. The equivalent baseband overall pulse is

$$p(t) = A \cos\left(\frac{\pi}{2\tau}\, t\right) \operatorname{rect}\left(t; \frac{3\tau}{2}\right), \qquad \text{with } \tau = 20\text{ms},\ A > 0.$$

Consider a simple receiver whose symbol-rate sampler is followed directly by a slicer.

a) Sketch the overall pulse $p(t)$ and determine the maximum bit rate for ISI-free transmission and the corresponding symbol duration T_S. This bit rate is to be used in what follows.

b) Sketch the transmission system in the equivalent discrete-time baseband domain.

c) How does the output of the symbol-rate sampler $q[k]$ depend on the transmit symbols $a[k]$?

d) At the slicer input, the filtered discrete-time noise is zero-mean, white, and complex Gaussian with variance N_0. The slicer uses the following decision rule:

$$\hat{a}[k] = \begin{cases} 1, & \operatorname{Re}\{q[k]\} > 0 \\ -1, & \operatorname{Re}\{q[k]\} < 0. \end{cases}$$

What is the symbol error probability of the receiver?

Problem 2 (20 credits)

A binary random variable $S \in \{0, 1\}$ (with $p_S(1) = 1/4$) is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a shifted Laplacian distribution

$$f_N(n) = a\, e^{-|n-\mu|}.$$

a) Calculate the factor a.

b) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

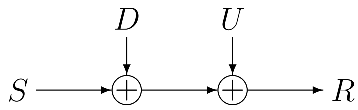
c) Calculate the parameter μ such that $\mathrm{P}\{N > 0\} = 1/3$. (This value of μ is to be used in what follows.)

d) Calculate and sketch the ML decision rule.

e) Calculate and sketch the MAP decision rule.

Problem 3 (20 credits)

The transmission of a symbol S from alphabet $\{-A/2, A/2\}$ with equal probabilities is corrupted by an interfering transmitter and by additive noise:



The interfering transmitter transmits symbols D from the alphabet $\{-A, 0\}$ with equal probabilities. The additive noise U is distributed as follows:

$$f_U(u) = \begin{cases} \dfrac{1}{A} - \dfrac{|u|}{A^2} & \text{if } |u| \leq A\,, \\ 0 & \text{else.} \end{cases}$$

Symbol S, interfering symbol D, and noise U are statistically independent.

a) Assume that there is no interfering transmitter, i.e., $R = S + U$. Calculate the decision rule and error probability of the ML receiver.

b) Express the interfering transmitter and the noise as a joint overall distortion $N = D + U$. Calculate and sketch the conditional distributions $f_{N|D}(n|d = -A)$ and $f_{N|D}(n|d = 0)$ as well as the unconditional distribution $f_N(n)$.

c) Calculate the error probability of the receiver derived in a) in the presence of the interfering transmitter, i.e., $R = S + D + U = S + N$.

d) Calculate the decision rule and error probability of the ML receiver in the presence of the interfering transmitter.

Problem 4 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{3z^2 + 10z + 3}{36z}.$$

Furthermore, the channel adds white noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of statistically independent symbols $a[k] \in \{1 + j, 1 - j, -1 + j, -1 - j\}$ with equal probabilities. Assume that the Viterbi algorithm is to be implemented at the output of the noise whitening filter. Sketch the state transition diagram and one elementary stage of the trellis diagram.