Digital Communications 1
Written exam on June 26, 2018

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Binary symbols $a[k] \in \{d/2, -d/2\}$ are transmitted by means of passband PAM over a dispersive channel with additive noise. The equivalent baseband overall pulse is

$$p(t) = \alpha \cos^2\left(\frac{\pi}{2\tau}t\right) \operatorname{rect}\left(t; \frac{3\tau}{2}\right), \qquad \text{with } \tau = 0.5\text{ms},\ \alpha > 0.$$

Consider a simple receiver whose symbol-rate sampler is followed directly by a slicer.

a) Sketch the overall pulse $p(t)$. Determine the minimum symbol duration T_s for ISI-free transmission and the corresponding bit rate. This symbol duration is to be used in what follows.

b) Sketch the transmission system in the equivalent discrete-time baseband domain.

c) How does the output $q[k]$ of the symbol-rate sampler depend on the transmit symbols $a[k]$? (Provide an equation.)

d) At the slicer input, the filtered discrete-time noise is zero-mean and complex Gaussian with variance N_0. The slicer uses the following decision rule:

$$\hat{a}[k] = \begin{cases} d/2 & \text{if } \operatorname{Re}\{q[k]\} > 0, \\ -d/2 & \text{if } \operatorname{Re}\{q[k]\} < 0. \end{cases}$$

What is the symbol error probability of the receiver?

Problem 2 (20 credits)

Consider equalization of a channel with equivalent discrete-time baseband pulse

$$p[k] = \delta[k] + \delta[k-1] + \delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1$.

a) Calculate $D_{\mathrm{ZF}}(e^{j\theta})$.

b) Now, assume that an MMSE equalizer $d_{\mathrm{MSE}}[k], k \in [-L, L]$ is used. What is a suitable value for L? Why?

c) Sketch the block diagram of an MMSE equalizer using L as determined in b).

d) Calculate $d_{\mathrm{MSE}}[k]$ for $k \in [-L, L]$.

Hint: The inverse of a matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

can be calculated as

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{13}a_{32} - a_{12}a_{33} & a_{12}a_{23} - a_{13}a_{22} \\ a_{23}a_{31} - a_{21}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{13}a_{21} - a_{11}a_{23} \\ a_{21}a_{32} - a_{22}a_{31} & a_{12}a_{31} - a_{11}a_{32} & a_{11}a_{22} - a_{12}a_{21} \end{pmatrix},$$

with

$$\det \mathbf{A} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

Problem 3 (20 credits)

A binary random variable $S \in \{-1, 1\}$ with $p_S(1) = 1/3$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & \text{if } n \geq b, \\ 0 & \text{if } n < b, \end{cases}$$

with a given $b \in \mathbb{R}$.

a) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

b) Calculate the parameter $b \in \mathbb{R}$ such that $\mathrm{P}\{N > 1\} = 1/2$. (This value of b is to be used in what follows.)

c) Calculate and sketch the ML decision rule.

d) Calculate and sketch the MAP decision rule.

Problem 4 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{3z^2 + 10z + 3}{3z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of statistically independent symbols $A[k] \in \{1, j, -1, -j\}$ with equal probabilities. Assume that the Viterbi algorithm is to be implemented at the output of the noise whitening filter. Sketch the state transition diagram.