Digital Communications 1

Written exam on December 11, 2014

Institute of Telecommunications

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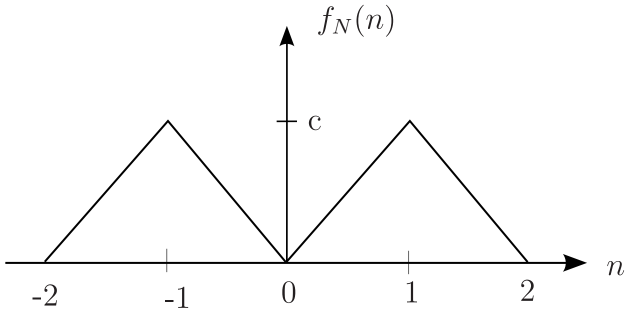
Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider transmission of a symbol $A \in \{-1, 0\}$ over a channel with additive Gaussian noise N, which is distributed as shown below.



The transmission probabilities are defined as $\mathrm{P}\{A = 0\} = p$.

a) Calculate the constant c and the MAP decision rule.

b) Calculate the ML decision rule and the corresponding symbol error probability.

c) In the following assume that the receiver knows the sign of the noise N and uses this additional information for detection.

c1) Assume $N > 0$. Sketch the conditional noise pdf for $N > 0$.

c2) Calculate the ML decision rule for $N > 0$ and for $N < 0$.

c3) How large is the symbol error probability of the ML detector that knows the sign of N?

Problem 2 (20 credits)

Over a discrete-time channel with impulse response $h[k] = \delta[k] - 0.6\,\delta[k-1]$, a sequence of symbols $a[k] \in \{-1, 0, 1\}$ is transmitted, with $a[k] = 0$ for $k < 0$. The additive noise is white and Gaussian. The received sequence $y[k]$ is given by $y[0] = 1.4$, $y[1] = 0.3$, and $y[2] = -1.3$.

a) Visualize this channel by a shift register circuit, a state diagram, and an elementary stage of the trellis diagram.

b) Use the Viterbi algorithm for ML sequence detection. Which sequence $\hat{A}[k]$ ($k = 0, 1, 2$) is obtained with this receiver?

c) An alternative receiver uses a zero-forcing-equalizer followed by a slicer. Which sequence $\hat{A}[k]$ ($k = 0, 1, 2$) is obtained with this receiver? You may assume that $y[k] = 0$ for $k < 0$.

Problem 3 (20 credits)

Consider the following pulse set:

$$g_1(t) = \operatorname{sinc}\left(\frac{5\pi}{3}\frac{t}{T_s}\right)\cos\left(\pi\frac{t}{T_s}\right)$$
$$g_2(t) = \operatorname{sinc}\left(\pi\frac{t}{T_s}\right)\cos\left(2\pi\frac{t}{T_s}\right)$$
$$g_3(t) = \operatorname{sinc}\left(\frac{\pi}{2}\frac{t}{T_s}\right)\cos\left(\frac{\pi}{2}\frac{t}{T_s}\right).$$

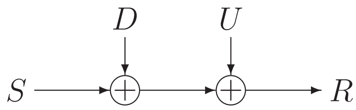
a) Sketch the pulses $g_i(t)$ in the frequency domain. Which pulses satisfy the Nyquist criterion? (A derivation is required.)

b) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion.

c) Using those pulses $g_i(t)$ that satisfy the generalized Nyquist criterion, consider orthogonal multipulse PAM with symbol alphabet $\mathcal{A} = \{-2, 2\}$. The symbols $A_m[k]$ are uniformly distributed and uncorrelated, i.e., $\mathrm{E}\{A_m[k]A_n^*[l]\} = \delta_{k,l}\delta_{m,n}$. Calculate and sketch the power spectral density $S_{\bar{S}}(j\omega)$ of the stationarized transmit signal $\bar{S}(t)$.

Problem 4 (20 credits)

The transmission of a symbol S from alphabet $\{-A/2, A/2\}$ with equal probabilities is corrupted by an interfering transmitter and by additive noise:



The interfering transmitter transmits a symbol D from the alphabet $\{-A, 0\}$ with equal probabilities. The additive noise U is distributed as follows:

$$f_U(u) = \begin{cases} \frac{3}{4A} - \frac{|u|}{2A^2} & \text{if } |u| \leq A\,, \\ 0 & \text{else.} \end{cases}$$

Symbol S, interfering symbol D, and noise U are statistically independent.

a) Assume that there is no interfering transmitter, i.e., $R = S + U$. Calculate the decision rule and error probability of the ML receiver.

b) Consider the interfering transmitter and the noise as a joint overall distortion $N = D + U$. Calculate and sketch the conditional probability density function $f_{N|D}(n|d = -A)$ and $f_{N|D}(n|d = 0)$ as well as the unconditional probability density function $f_N(n)$.

c) Calculate the error probability of the receiver derived in a) in the presence of the interfering transmitter, i.e., $R = S + D + U = S + N$.

d) Calculate the decision rule and error probability of the ML receiver in the presence of the interfering transmitter.