Digital Communications 1
Written exam on January 25, 2017

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A stationary and white bit sequence (bit rate $R_b = 64\,\text{kbit/s}$) is to be transmitted over a bandlimited AWGN channel (power spectral density $S_N(j\omega) = N_0/2 = 9 \cdot 10^{-7}\,\text{W/Hz}$, bandwidth $B_\text{T} = 24\,\text{kHz}$). The modulation scheme is passband PAM using an M_a-ary QAM constellation. The Fourier transform of the transmit pulse is given by $G(j\omega) = \sqrt{R(j\omega)}$, where $R(j\omega)$ is the Fourier transform of a raised-cosine pulse with roll-off factor $\alpha = 0.5$.

a) Determine the size of the symbol alphabet M_a and sketch the signal constellation in the complex plane.

b) Specify a receive filter for ISI-free transmission.

c) Sketch a block diagram of the ML receiver.

For the following, assume that symbols with positive real part are twice as likely than symbols with negative real part.

d) Calculate the minimum symbol distance d_a for a symbol error probability of $\text{P}\{\mathcal{E}_s\} = 10^{-7}$. Use for this the approximation $\text{P}\{\mathcal{E}_s\} \approx \overline{\mathcal{N}}\,\mathcal{Q}\left(\frac{d_\text{min}}{\sqrt{2}\sigma_Z}\right)$.

e) How does the original signal constellation need to be shifted to minimize the mean transmit power $P_{\bar{S}}$? Sketch the shifted constellation. How large is the achieved reduction of $P_{\bar{S}}$?

Problem 2 (20 credits)

Consider passband PAM transmission of symbols $A[k]$ that are taken from the alphabet $\{2, -2\}$ with equal probabilities. The symbol sequence $A[k]$ and the equivalent discrete-time noise $Z[k]$ are independent and both white. The noise is zero-mean and circularly symmetric complex Gaussian with variance $\sigma_Z^2 = 1/4$. The receiver consists of a receive filter and a slicer with decision threshold 0. The equivalent discrete-time baseband pulse is

$$p[k] = \delta[k] - \frac{1}{2}\delta[k-1].$$

a) Provide an expression of the signal $Q[k]$ at the slicer input and indicate the components constituting 1) the desired signal, 2) the ISI, and 3) the additive noise.

b) Find the likelihood function $f_{Q[k]|A[k],A[k-1]}(q[k]|a[k], a[k-1])$ and calculate the conditional error probabilities $\mathrm{P}\{\hat{A}[k] \neq A[k]|A[k], A[k-1]\}$ for all four cases $(A[k], A[k-1]) \in \{2, -2\}^2$. Calculate the unconditional error probability $\mathrm{P}\{\hat{A}[k] \neq A[k]\}$.

c) Calculate the mean powers of the three signal components from a).

Consider an extended receiver using a decision feedback equalizer that minimizes the mean-square error at the slicer input. The coefficients of the feedforward filter and of the feedback filter are $\mathbf{d}_{\mathrm{MSE}} = \frac{8}{293}(-1 \;\; 34 \;\; 0)^T$ and $\mathbf{v}_{\mathrm{MSE}} = \frac{136}{293}(-1 \;\; 0)^T$, respectively.

d) Specify the signal at the slicer input and indicate the components constituting 1) the desired signal, 2) the ISI, and 3) the additive noise (assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).

Problem 3 (20 credits)

A binary random variable $S \in \{-1, 1\}$ with $p_S(1) = 1/3$ is corrupted by additive noise $N \in \mathbb{R}$ that is statistically independent of S and has a modified exponential distribution

$$f_N(n) = \begin{cases} ae^{-n} & \text{if } n \geq b, \\ 0 & \text{if } n < b, \end{cases}$$

with a given $b \in \mathbb{R}$.

a) Calculate the probabilities $\mathrm{P}\{N < n_1\}$ and $\mathrm{P}\{N > n_2\}$.

b) Calculate the parameter $b \in \mathbb{R}$ such that $\mathrm{P}\{N > 1\} = 2/3$. (This value of b is to be used in what follows.)

c) Calculate and sketch the ML decision rule.

d) Calculate and sketch the MAP decision rule.

Problem 4 (20 credits)

Consider a passband PAM system with an ML sequence detector. In the equivalent discrete-time baseband domain, the channel is described by the folded spectrum

$$S_h(z) = \frac{3z^2 + 10z + 3}{3z}.$$

Furthermore, the channel adds white Gaussian noise.

a) Sketch the function $S_h(e^{j\theta})$ for $\theta \in [-\pi, \pi]$.

b) Find the poles and zeros of $S_h(z)$.

c) Find a minimum-phase factorization of $S_h(z)$.

d) Calculate the transfer function and impulse response of the equivalent discrete-time system including the noise whitening filter.

e) Consider the transmission of symbols $A[k] \in \{1, j, -1, -j\}$. Assume that the Viterbi algorithm is to be implemented at the output of the noise whitening filter. Sketch the state transition diagram.