Digital Communications 1
Written exam on December 7, 2017

Institute of Telecommunications

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Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A stationary and white bit sequence (bit rate $R_b = 64\,\text{kbit/s}$) is to be transmitted over a bandlimited AWGN channel (power spectral density $S_N(j\omega) = N_0/2 = 9 \cdot 10^{-7}\,\text{W/Hz}$, bandwidth $B_\text{T} = 24\,\text{kHz}$). The modulation scheme is passband PAM using an M_a-ary QAM constellation. The Fourier transform of the transmit pulse is given by $G(j\omega) = \sqrt{R(j\omega)}$, where $R(j\omega)$ is the Fourier transform of a raised-cosine pulse with roll-off factor $\alpha = 0.5$.

a) Determine the size of the symbol alphabet M_a and sketch the corresponding symbol constellation.

b) Design a receive filter for ISI-free transmission.

c) Sketch the block diagram of the ML receiver.

For the following, assume that symbols with positive real part are twice as likely than symbols with negative real part.

d) Calculate the minimum symbol distance d_a for a symbol error probability of $\mathrm{P}\{\mathcal{E}_s\} = 10^{-7}$. Use for this the approximation $\mathrm{P}\{\mathcal{E}_s\} \approx \overline{\mathcal{N}}\mathcal{Q}\left(\frac{d_\text{min}}{\sqrt{2}\sigma_Z}\right)$.

e) How does the symbol constellation need to be shifted to minimize the mean transmit power $P_{\bar{S}}$? Sketch the shifted constellation. How large is the achieved reduction of $P_{\bar{S}}$?

Problem 2 (20 credits)

Consider a passband PAM system with received pulse

$$h(t) = e^{-ct}\mathrm{u}(t), \qquad c > 0,$$

where $\mathrm{u}(t)$ denotes the unit step function. The additive noise is white with power spectral density $S_N(j\omega) = N_0/2$. The receiver consists of a receive filter $f(t)$, followed by a symbol-rate sampler and a zero-forcing (ZF) linear equalizer.

a) Assume that $f(t) = \delta(t)$, i.e., the overall complex baseband pulse is $p(t) = h(t)$.

- **a1)** Calculate the equivalent discrete-time pulse $p[k]$ at the input of the equalizer. (*Hint*: Use the discrete-time unit step function $\mathrm{u}[k]$.) Determine the variance σ_Z^2 of the equivalent discrete-time noise $Z[k]$.
- **a2)** Calculate the transfer function $D_{\mathrm{ZF}}(e^{j\theta})$ and the impulse response $d_{\mathrm{ZF}}[k]$ of the ZF linear equalizer.
- **a3)** Assume that $S_Z(e^{j\theta}) \equiv 1$. Determine the power spectral density $S_U(e^{j\theta})$ of the noise $U[k]$ at the output of the ZF linear equalizer.

b) Consider the receive filter $f(t)$ that maximizes the SNR at the input of the ZF linear equalizer.

- **b1)** Determine $f(t)$ and the transfer function $F(j\omega)$.
- **b2)** Calculate the equivalent discrete-time pulse $p[k]$ at the input of the equalizer.
- **b3)** Calculate the transfer function $D_{\mathrm{ZF}}(e^{j\theta})$ and the impulse response $d_{\mathrm{ZF}}[k]$ of the ZF linear equalizer.

Problem 3 (20 credits)

Two signals $x(t)$ and $y(t)$ are used for binary transmission over an AWGN channel. Both $x(t)$ and $y(t)$ are taken from the set $\{s^{(1)}(t), s^{(2)}(t), s^{(3)}(t)\}$, where

$$\begin{aligned} s^{(1)}(t) &= \alpha_1 \left(1 - \frac{t^2}{T^2}\right) \operatorname{rect}(t;T) \\ s^{(2)}(t) &= \alpha_2 \left(1 - \frac{t^2}{T^2}\right) \operatorname{rect}(t;T/2) \\ s^{(3)}(t) &= \alpha_3 \operatorname{rect}(t;T) \end{aligned}$$

with $\alpha_1, \alpha_2, \alpha_3 > 0$. The receiver performs ML detection.

a) Find the values of α_1, α_2, and α_3 such that $\|s^{(1)}\|^2 = \|s^{(2)}\|^2 = \|s^{(3)}\|^2 = 1$. These values are to be used in what follows.

b) How does the error probability of the binary ML detector depend on $\|x\|^2$, $\|y\|^2$ and $\langle x, y\rangle$?

c) Find the two signals among $\{s^{(1)}(t), s^{(2)}(t), s^{(3)}(t)\}$ that minimize the error probability. These two signals are to be used as $x(t)$ and $y(t)$ in what follows.

d) One of the transmit signals ($x(t)$ or $y(t)$) is multiplied by a constant phase factor $e^{j\phi}$, where $\phi \in [0, 2\pi)$. Find the value of ϕ that minimizes the error probability.

Problem 4 (20 credits)

Consider an AWGN channel with noise power spectrum $N_0/2 = 1.295 \cdot 10^{-5}$W/Hz and transmission of M_a-ary symbols using orthogonal multipulse modulation. The transmit pulses are assumed to satisfy the generalized Nyquist criterion; their energy is $E_g = 7 \cdot 10^{-4}$J. The symbols are detected by means of the ML sequence detector.

a) Using the nearest neighbor approximation, calculate the maximum number of symbols M_a such that a symbol error probability of less than $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\} = 1.55 \cdot 10^{-6}$ is achieved. This value of M_a is to be used in what follows.

b) Calculate the resulting symbol error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_s\}$ (again using the nearest neighbor approximation) and the bit error probability $\mathrm{P}_{\mathrm{ML}}\{\mathcal{E}_b\}$.

c) Calculate the maximum spectral efficiency $\nu_{\max}$, assuming bandpass transmission.