Modulations- und Detektionsverfahren

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Please note:

- You may use the lecture notes, a pocket calculator and a *mathematical* table of formulas of your choice.
- Personal notes, material from exercise classes, and pre-computed problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- State your derivations in detail. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider equalization of a channel using a decision feedback equalizer. The equivalent discrete-time baseband pulse is given by

$$p[k] = \delta[k] + \frac{1}{2}\delta[k-1] + \frac{1}{2}\delta[k+1].$$

The transmit symbols are taken from the alphabet $\{1, -1\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise are uncorrelated and both white. The noise is zero-mean with variance $\sigma_Z^2 = 1/4$.

a) Assume a general feedforward filter $d[k]$, $k \in [-L, L]$. Calculate the equivalent overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

b) Consider the case $L = 0$.

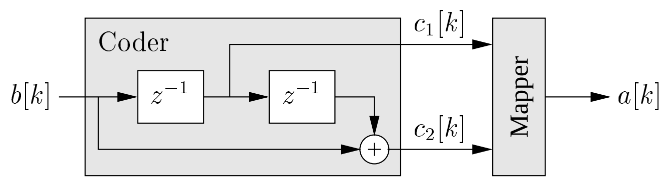
- **b1)** What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous decisions were correct)?
- **b2)** Using this minimum length K, calculate the coefficients of the feedforward and feedback filters which minimize the MSE at the slicer input.

c) Consider now the case $L = 1$.

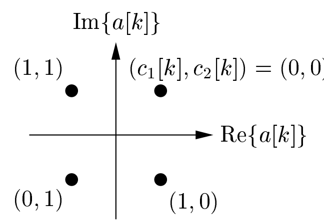
- **c1)** What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous decisions were correct)?
- **c2)** The coefficients of the feedforward filter are $\mathbf{d}_{\mathrm{MSE}} = (1/17) \cdot (-2,\, 10,\, 8)^{\mathrm{T}}$. Calculate the coefficients of the feedback filter (of length K, as in **c1)**) which minimize the MSE at the slicer input.
- **c3)** Specify the signal at the slicer input and indicate the components which constitute 1.) the desired signal, 2.) ISI and 3.) additive noise (still assuming that all previous decisions were correct).
- **c4)** Calculate the mean powers of the three signal components from **c3)**.

Problem 2 (20 credits)

Consider a system with trellis coded modulation (TCM). The transmitter consists of a convolutional encoder and a symbol mapper (as illustrated below).



TCM-Sender



Mapper

The convolutional encoder uses the data bits $b[k]$ to produce two code bits at each step, $c_1[k] = b[k-1]$ and $c_2[k] = b[k] \oplus b[k-2]$ ($\oplus$ denotes addition modulo 2). The symbol mapper maps these code bits to symbols $a[k]$ (with $|a[k]|^2 = 2$).

The received signal is given by $y[k] = a[k] + w[k]$ (ISI-free channel). The complex white noise $w[k]$ has a Laplace distribution, i.e.,

$$f_w(\xi) = \frac{1}{2} e^{-|\xi|}.$$

a) For the bit sequence $\{1, 0, 1, 1, 0\}$, derive the corresponding code bits $c_1[k]$, $c_2[k]$ and transmit symbols $a[k]$ (assuming that all previous data bits $b[k]$ are zero).

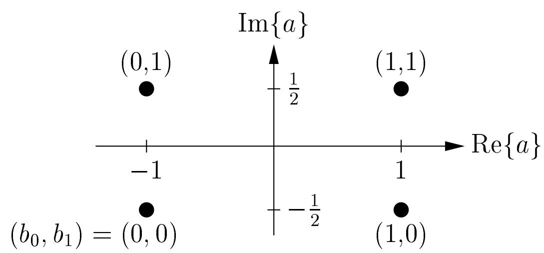
b) Sketch a trellis diagram of the TCM transmitter.

c) The transmitted bit sequence is detected by an ML receiver that is implemented using the Viterbi algorithm. Derive an expression of the path metric of the Viterbi algorithm.

d) Using the Viterbi algorithm (with the above metric), calculate the detected bits $\hat{b}[k]$ from the received signal $y[0] = 1.1 - j1.4$, $y[1] = 0.8 + j1.2$, $y[2] = -0.7 - j0.9$ (assuming that $b[k] = 0$ for $k < 0$).

Problem 3 (20 credits)

In a passband PAM system, two bitstreams $b_0[k]$ and $b_1[k]$ are to be transmitted with unequal reliabilities. The bits are mapped to symbols $a[k]$ according to the following figure:



All transmit bits are statistically independent and assume the values 0 and 1 with equal probabilities. The Fourier transform of the transmit pulse $g(t)$ is given by

$$G(j\omega) = \sqrt{3T_s}\,\cos\left(\frac{T_s}{4}\omega\right)\,\mathrm{rect}\left(\omega, \frac{2\pi}{T_s}\right).$$

The channel is distortionless with additive white Gaussian noise (with power spectral density $N_0/2 = 0.04\,\mathrm{W/Hz}$). A sampled matched filter at the receiver front-end yields the received sequence $q[k] = p[k] * a[k] + z[k]$.

a) Explain why the bit streams $b_0[k]$ and $b_1[k]$ are transmitted with unequal reliabilities.

b) Calculate the impulse response $p[k]$ and the transfer function $P(e^{j\theta})$ of the equivalent discrete-time channel (including the sampled matched filter). Calculate the power spectral density of the noise at the sampler output, $S_Z(e^{j\theta})$.

c) Derive the decision rule of the MAP receivers for $b_0[k]$ and $b_1[k]$ and sketch the respective decision regions.

d) Calculate exact expressions for the bit error probabilities $\mathrm{P}\{\mathcal{E}_{b,i}\} = \mathrm{P}\{\hat{b}_i[k] \neq b_i[k]\}$.

Problem 4 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-1, 1, -1+2j, 1+2j\}$. The symbols are assumed white and uniformly distributed.

a) Calculate the symbol power P_A. How must the symbol alphabet be shifted to minimize the symbol power?

In the following, the shifted symbol alphabet is to be used.

b) For transmission, the symbol sequence $a[k]$ is transformed into a sequence $b[k] = a[k] + \alpha a[k-1]$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transformed symbols $B[k]$.

c) The equivalent baseband transmit signal is $s_{\text{LP}}(t) = \sum_{k=-\infty}^{\infty} b[k] g(t - kT_s)$. Calculate the power spectral density $S_{\bar{S}_{\text{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\text{LP}}(t)$.

d) The spectrum $S_{\bar{S}_{\text{LP}}}(j\omega)$ is to be zero at frequency $\omega = \frac{\pi}{T_s}$. Which choice of α satisfies this condition?

e) Assume that the transmit pulse $g(t)$ is a sinc pulse: $g(t) = \text{sinc}(\frac{\pi t}{T_s})$. Sketch the power spectral density of the stationarized transmit signal $S_{\bar{S}_{\text{LP}}}(j\omega)$ for this case, using α from **d)**.