Digital Communications 1
Written exam on June 29, 2022

Institute of Telecommunications

TU Wien

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

Consider a passband PAM system with symbol alphabet $\mathcal{A} = \{-2, 2, -4j\}$. All three symbols are equally likely. The symbols $A[k]$ are assumed statistically independent.

a) Calculate the symbol power P_A. How should the symbol alphabet be shifted in order to minimize the symbol power?

In the following, use the shifted symbol alphabet.

b) Prior to transmission, the symbol sequence $A[k]$ is transformed into the sequence $B[k] = A[k] - \alpha A[k-2]$ with $\alpha \in \mathbb{R}$. Calculate the power spectral density $S_B(e^{j\theta})$ of the transmitted symbols $B[k]$.

c) The transmit signal in the equivalent baseband domain is given by $S_{\mathrm{LP}}(t) = \sum_{k=-\infty}^{\infty} B[k] g(t - kT_s)$, where $g(t)$ is the Nyquist pulse with minimum bandwidth for a given T_s. Calculate the power spectral density $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ of the stationarized transmit signal $\bar{S}_{\mathrm{LP}}(t)$.

d) Determine α such that $S_{\bar{S}_{\mathrm{LP}}}(j\omega)$ is zero at the frequency $\omega = \frac{\pi}{2T_s}$.

Problem 2 (20 credits)

Consider transmission of a symbol A from the alphabet $\mathcal{A} = \{-\frac{d}{4}, \frac{3d}{4}\}$ with $p_A(-\frac{d}{4}) = \frac{3}{4}$. In the receiver, the random variable Q at the slicer input contains real-valued additive noise Z, i.e.,

$$Q = A + Z.$$

The noise has a triangular distribution given by

$$f_Z(z) = \frac{1}{\epsilon}\left(1 - \frac{|z|}{\epsilon}\right) \mathrm{rect}(z; \epsilon)$$

with parameter $\epsilon > 0$.

a) Sketch $f_Z(z)$.

b) For the following assume $\epsilon = d/2$.

b1) Sketch the distribution $f_{Q|A}(q|a)p_A(a)$ for each $a \in \mathcal{A}$.

b2) Determine the MAP decision rule and sketch the decision intervals. Calculate the symbol error probability of the MAP detector.

c) Repeat part b) under the assumption $\epsilon = d$.

Problem 3 (20 credits)

A sequence of symbols $a[k] \in \{-1, 0, 1\}$, with $a[k] = 0$ for $k < 0$, is transmitted over a discrete-time channel. This channel is dispersive with impulse response $h[k] = \delta[k] - 0.4\,\delta[k-1]$ and has additive noise that is white and Gaussian. The received sequence $y[k]$ is given by $y[0] = 1.3$, $y[1] = 0.4$, and $y[2] = -1.2$.

a) Visualize this channel by a shift register circuit, a state diagram, and an elementary stage of the corresponding trellis diagram.

b) Use the Viterbi algorithm for ML sequence detection. Which sequence $\hat{a}[k]$ $(k = 0, 1, 2)$ is obtained with this receiver?

c) An alternative receiver uses a zero-forcing equalizer followed by a slicer. Which sequence $\hat{a}[k]$ $(k = 0, 1, 2)$ is obtained with this receiver? You may assume that $y[k] = 0$ for $k < 0$.

Problem 4 (20 credits)

Consider the pulse set defined as

$$g_m(t) = c(t)\cos\left(\pi\left(m + \frac{1}{2}\right)\frac{t}{T_s}\right), \quad m = 1, 2, \ldots, 8, \tag{1}$$

where $c(t)$ is such that $c(t) * c(-t)$ is a Nyquist pulse for the *double* symbol period $2T_s$, i.e., $\int_{-\infty}^{\infty} c(t)c(t - k2T_s)\mathrm{d}t = \delta[k]$, and $\mathrm{Re}\{C(j\omega)C(j(\frac{\pi}{T_s} - \omega))\} = 0$ for $0 \le \omega \le \frac{\pi}{2T_s}$. Also, $C(j\omega)$ is bandlimited to $\frac{\pi}{T_s}$.

a) Show that this pulse set satisfies the generalized Nyquist criterion.

b) What is the minimum bandwidth of $c(t)$?

c) Sketch the pulses $g_m(t)$ in the frequency domain.

Consider the transmission of 8-ary symbols by means of orthogonal multipulse modulation using the pulse set defined by (1). The transmit pulses have energy $E_g = 2 \cdot 10^{-4}$ J. The channel is AWGN with noise power spectrum $N_0/2 = 4 \cdot 10^{-5}$ W/Hz. The ML sequence detector is used.

d) Calculate a simple approximation for the resulting symbol error probability.

e) Calculate the corresponding (approximate) bit error probability.