Modulations- und Detektionsverfahren

Written exam on March 15, 2012

Institute of Telecommunications

Vienna University of Technology

Please note:

- You may use the official lecture notes, a pocket calculator, and a collection of mathematical formulas.
- Personal notes, materials from exercise classes, and pre-calculated problems may not be used.
- Legible writing and a clear layout of your derivations and solutions are absolutely necessary!
- Provide detailed derivations. When using results from the lecture notes, they must be explicitly referenced.

Problem 1 (20 credits)

A transmission system uses the two transmit signals

$$s_1(t) = \sin(\omega_0 t)\ \mathrm{rect}(t - T; T)$$
$$s_2(t) = \sin(\omega_0 t + \theta)\ \mathrm{rect}(t - T; T),$$

with $T = \pi/\omega_0$ and $\omega_0 > 0$.

a) Calculate $\langle s_1, s_2 \rangle$ and sketch it as a function of θ.

b) Calculate $\|s_1\|$, $\|s_2\|$, and $\|s_1 - s_2\|$.

Consider transmission of $s_i(t)$ with $i \in \{1, 2\}$ over an AWGN channel:

$$y(t) = s_i(t) + n(t),$$

where $y(t)$ is the received signal and the noise $n(t)$ has a power spectral density of $N_0/2$.

c) Find the decision rule of the ML detector $\hat{i}_{\mathrm{ML}}(y)$. Sketch an implementation of the ML detector that uses only one inner product.

d) Using your results from b), calculate the error probability of the ML detector.

Problem 2 (20 credits)

Consider passband PAM transmission using the QPSK signal constellation $\mathcal{A} = \{1, j, -1, -j\}$. The transmission is ISI-free with complex gain factor $p[0] = 1 + j$. The receiver performs symbolwise ML detection. The real part and the imaginary part of the noise at the slicer input, $z[k] = z_{\mathrm{R}}[k] + j z_{\mathrm{I}}[k]$, are statistically independent zero-mean Gaussian random variables with variances $\sigma_{\mathrm{R}}^2 = 1$ and $\sigma_{\mathrm{I}}^2 = 0.25$, respectively.

a) Sketch the signal constellation.

b) Sketch the signal constellation as seen at the slicer input and the decision regions of the symbolwise ML detector.

c) Assume that the symbol -1 was transmitted. Calculate the probabilities of detecting 1, j, and $-j$. Calculate the conditional symbol error probability for the case that -1 was transmitted.

d) Calculate the conditional symbol error probabilities for the cases that 1, j, and $-j$ were transmitted. Calculate the unconditional symbol error probability.

e) Assume that the transmitter decides to use only the symbols 1 and -1 but the receiver is not changed. How does this influence the unconditional symbol error probability?

Problem 3 (20 credits)

Consider the following pulse set:

$$g_1(t) = \text{sinc}\left(\frac{\pi}{2}\frac{t}{T_s}\right)\cos\left(\frac{5\pi}{2}\frac{t}{T_s}\right)$$
$$g_2(t) = \text{sinc}\left(\frac{\pi}{2}\frac{t}{T_s}\right)\cos\left(\pi\frac{t}{T_s}\right)$$
$$g_3(t) = g_2(2t) - g_1(t)$$
$$g_4(t) = g_1(2t).$$

a) Sketch the pulses in the frequency domain.

b) Which of the pulses satisfy the Nyquist criterion? (A derivation is required.)

c) Show that those pulses $g_i(t)$ that satisfy the Nyquist criterion also satisfy the generalized Nyquist criterion.

Problem 4 (20 credits)

Consider equalization of a channel using a decision feedback equalizer. The equivalent discrete-time baseband pulse is given by

$$p[k] = \delta[k] - \frac{1}{2}\delta[k-1] - \frac{1}{2}\delta[k+1].$$

The transmit symbols are taken from the alphabet $\{2, -2\}$ with equal probabilities. The symbol sequence and the equivalent discrete-time noise $Z[k]$ are uncorrelated and both white. The noise is zero-mean, and the noise variance is $\sigma_Z^2 = 1/4$.

a) Assume a general feedforward filter $d[k], k \in [-L, L]$. Calculate the equivalent overall pulse $p^{(d)}[k]$ at the output of the feedforward filter. How long is this pulse?

b) Consider now the case $L = 1$. What is the minimum length K of the feedback filter such that as much ISI as possible is canceled (under the assumption that all previous symbol decisions were correct)?

c) The coefficients of the feedforward filter are $\mathbf{d}_{\text{MSE}} = (8/433)\cdot(11\ 34\ {-41})^T$. Calculate the coefficients of the feedback filter (of length K, as calculated in b)) which minimize the MSE at the slicer input.

d) Specify the signal at the slicer input and indicate the components which constitute 1) the desired signal, 2) ISI, and 3) additive noise (still assuming that all previous symbol decisions were correct).

e) Calculate the mean powers of the three signal components from d).