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Übungen zur Vorlesung
Einführung in das Programmieren für TM

Serie 6

Aufgabe 6.1. Write a function `maxabs` which returns the very first entry x_j with largest absolute value of a given vector $x \in \mathbb{R}^n$, i.e., x_j satisfies $|x_j| = \max\{|x_i| \, : \, i = 1, \dots, n\}$ and if $|x_i| = |x_j|$, then it holds $i \geq j$. Additionally, write a main program that reads in the vector x and calls the function `maxabs`. The vector x should be realized as a static array, where the length is given by a constant in the main program, but the function `maxabs` should be implemented for arrays with arbitrary lengths. Save your source code as `maxabs.c` into the directory `serie06`.

Aufgabe 6.2. Let the two series

$$a_N := \sum_{n=0}^{N} \frac{1}{(n+1)^2} \quad \text{und} \quad b_M := a_M^2 = \sum_{m=0}^{M} \sum_{k=0}^{m} \frac{1}{(k+1)^2(m-k+1)^2}$$

be given. Write a program that measures the time used for the computation of a_N resp. b_M for different values of N resp. M. Print out the results tabularly. Do the results meet your expectations? Save your source code as `timing.c` into the directory `serie06`. *Hint:* Think of the computational complexity (Aufwand) for the computation of a_N resp. b_M.

Aufgabe 6.3. Write a function `lcm` that computes the *least common multiple* of two given natural numbers $a, b \in \mathbb{N}$. For the solution, you can either compute the prim factors of both numbers or use the relation $ab = \gcd(a, b) \cdot \mathrm{lcm}(a, b)$, where $\gcd(a, b)$ denotes the *greatest common divisor*. Save your source code as `lcm.c` into the directory `serie06`.

Aufgabe 6.4. Write a `void`-function `dec2bin`, which, given a natural number $0 \leq z < 256$, computes and displays its representation in the binary numeral system. First, the program has to determine the coefficients $a_i \in \{0, 1\}$, $i = 0, \dots, 7$, such that $z = \sum_{i=0}^{7} a_i 2^i$. Then, the binary representation should be visualized in a suitable way. For instance, for $z = 77$ the function prints out the sequence `0 1 0 0 1 1 0 1`. Moreover, write a main program, which reads z from the keyboard and calls the function `dec2bin`. Save your source code as `dec2bin.c` into the directory `serie06`.

Aufgabe 6.5. The quotient sequence $(a_{n+1}/a_n)_{n \in \mathbb{N}}$ corresponding to the Fibonacci sequence $(a_n)_{n \in \mathbb{N}}$,

$$a_0 := 1, \quad a_1 := 1, \quad a_n := a_{n-1} + a_{n-2} \quad \text{für } n \geq 2,$$

converges towards the *golden ratio* $(1 + \sqrt{5})/2$. In particular, the difference sequence

$$b_n := \frac{a_{n+1}}{a_n} - \frac{a_n}{a_{n-1}}$$

converges towards 0. Write a function `cauchy` that returns, for given $k \in \mathbb{N}$, the smallest $n \in \mathbb{N}$ such that $|b_n| \leq 1/k$. Moreover, write a main program that reads in $k \in \mathbb{N}$ and prints out the index $n \in \mathbb{N}$. Save your source code as `goldenRatio.c` into the directory `serie06`.

Aufgabe 6.6. The sine function can be represented as a series via

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

The corresponding n-th partial sum is given by

$$S_n(x) = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

Write a function sin_, which, given $x \in \mathbb{R}$ and $\varepsilon > 0$, returns the first value of $S_n(x)$ such that

$$|S_n(x) - S_{n-1}(x)|/|S_n(x)| \le \varepsilon \quad \text{or} \quad |S_n(x)| \le \varepsilon.$$

Then, write a main program, which reads $x \in \mathbb{R}$ and $\varepsilon > 0$ from the keyboard, calls the function and displays the computed value $S_n(x)$, as well as the value $\sin(x)$, the absolute error $|S_n(x) - \sin(x)|$ and the relative error $|S_n(x) - \sin(x)|/|\sin(x)|$ (provided $\sin(x) \neq 0$). Save your source code as `sin.c` into the directory `serie06`.

Aufgabe 6.7. You place your money with your trusted bank for a fixed annual percentage rate. Write a function `capital` which computes your capital after $n \in \mathbb{N}$ years for a fixed annual percentage p (in percent %), and your starting capital $x \in \mathbb{R}_{\geq 0}$. The function should print out your money as follows

```
Year    Capital
====    =======
0      1000.00
1      1010.00
2      1020.10
3      1030.30
..      .......
10      1104.62
```

For this example holds $p = 1$, $n = 10$, and $x = 1000.00$. Furthermore, write a function `runtime` which computes how long (at least) you have to wait to increase your starting capital x to $x_{\max}$ for a fixed percentage p. The function reads in x, p, and $x_{\max}$. Additionally, write a main program that tests both functions. How long does it take to be a millionaire, if you invest $x = 1000$ with a fixed percentage $p = 4$? Save your source code as `capital.c` into the directory `serie06`.

Aufgabe 6.8. Which types of comments do you know? What is the output of the following code and why?

```
#include <stdio.h>

/*int f(double x) {
    return (int) x;
  }
*/

main() {
  int x = 4;
  int y = 2*x*/* f(0.1)+3
               */1/4;
  // y = 1;
  printf("y = %d\n",y); // Print out result
}
```