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Sommersemester 2016
25. Mai 2016

# Übungen zur Vorlesung Computermathematik

## Serie 10

**Aufgabe 10.1.** Write a LaTeX-file which consists of this exercise sheet– headline of the sheet up to and including Aufgabe 10.2. To generate a LaTeX-command `\command`, you can use `\verb|\command|`.

**Aufgabe 10.2.** Write a text of your choice with headline and at least 400 words in LaTeX. Use 12pt as font size. The text should consist of at least two paragraphs. What does the warning `Overfull hbox` mean? If necessary, modify the text such that LaTeX does not return this warning. Take a look at the generated log-file `name.log` and prepare to be able to explain the content of the file during your exercise. How would one have to modify the text in order to avoid the warning `Overfull hbox`? Add the reference from where you took your text in a footnote `\footnote{...}`.

**Aufgabe 10.3.** If one writes the command `\renewcommand{\familydefault}{\sfdefault}` at the head of a LaTeX-document, LaTeXchooses the font "Arial" instead of the standard font "Times New Roman". Modify your file `text.tex` from the previous exercise such that the standard font is different and use $1,5$ line spacing. Divide your text in at least two sections and add a table of contents. What is the automatically generated file `arial.toc` good for? Why does one have to compile multiple times with LaTeX until the table of contents is generated correctly?

**Aufgabe 10.4.** Let $I$ be a nonempty open interval. Then it holds for $f, g \in C^\infty(I)$ and $n \in \mathbb{N}$

$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}.$$

Write a LaTeX-file which includes the assertion and the (detailed) proof of the product rule for the $n$-th derivative.

**Aufgabe 10.5.** Write the following text in LaTeX, where the symbol $\pm$ is generated by `\pm`: For given ***basis*** $b \in \mathbb{N}_{\geq 2}$, ***mantissa length*** $t \in \mathbb{N}$ and ***exponential bounds*** $e_{\min} < 0 < e_{\max}$ we define the set of ***normalized floating point numbers*** $\mathbb{F} := \mathbb{F}(b, t, e_{\min}, e_{\max}) \subset \mathbb{R}$ by

$$\mathbb{F} = \{0\} \cup \Big\{ \Big(\sigma \sum_{k=1}^{t} a_k b^{-k}\Big) b^e \,\Big|\, \sigma \in \{\pm 1\}, a_j \in \{0, \dots, b-1\}, a_1 \neq 0, e \in \mathbb{Z}, e_{\min} \leq e \leq e_{\max} \Big\}.$$

The finite sum $a = \sum_{k=1}^{t} a_k b^{-k}$ is called ***normalized mantissa*** of a floating point number.

**Aufgabe 10.6.** Write the following definition of an upper triangle matrix:

$$A := \begin{pmatrix} \alpha & 2\alpha & 3\alpha & \cdots & n\alpha \\ 0 & \alpha & 2\alpha & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 3\alpha \\ \vdots & \ddots & \ddots & \ddots & 2\alpha \\ 0 & \cdots & 0 & 0 & \alpha \end{pmatrix} \in \mathbb{R}^{n \times n}_{\mathrm{tria}}$$

in LaTeX. The dots are generated by `\cdots`, `\vdots` and `\ddots`, the symbol $\times$ by `\times`.

**Aufgabe 10.7.** Write the following formula in a LaTeX-file: Let $f, g : \mathbb{R} \to \mathbb{R}$ be given functions given by

$$f(x) := \begin{cases} -1 & \text{if } x < -\frac{\pi}{2}, \\ \sin(x) & \text{if } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \\ 1 & \text{if } x > \frac{\pi}{2}. \end{cases} \qquad \text{and} \qquad g(x) := \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

**Aufgabe 10.8.** Write a LaTeX-file in which the following theorem of Brezzi is formulated. Define suitable macros for the norms as well as the bilinear forms $a(\cdot, \cdot)$ und $b(\cdot, \cdot)$.

**Theorem (Brezzi 1974).** Let $X$ and $Y$ be hilbert spaces. Further, let $a : X \times X \to \mathbb{R}$ and $b : X \times Y \to \mathbb{R}$ be continuous bilinear forms and $X_0 := \big\{ x \in X \,:\, b(x, \cdot) = 0 \in Y^* \big\}$. Under the assumptions

- $\alpha := \inf_{v \in X_0 \setminus \{0\}} \dfrac{a(v,v)}{\|v\|_X^2} > 0$, i.e., $a(\cdot, \cdot)$ is elliptic auf $X_0$,
- $\beta := \inf_{y \in Y \setminus \{0\}} \sup_{x \in X \setminus \{0\}} \dfrac{b(x,y)}{\|x\|_X \|y\|_Y} > 0$.

there holds the assertion: For each $(x^*, y^*) \in X^* \times Y^*$ there is a unique solution $(x, y) \in X \times Y$ of the so-called saddle point problem

$$\begin{array}{lclll} a(x, \widetilde{x}) & + & b(\widetilde{x}, y) & = & x^*(\widetilde{x}) & \text{for all } \widetilde{x} \in X, \\ b(x, \widetilde{y}) & & & = & y^*(\widetilde{y}) & \text{for all } \widetilde{y} \in Y. \end{array} \tag{1}$$