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Übungen zur Vorlesung Computermathematik

Serie 11

Aufgabe 11.1. Three natural numbers $a, b, c \in \mathbb{N}$ are called *Pythagorean triple*, if $a^2+b^2 = c^2$. Prove via the approach $a := m^2 - n^2$ and $b := 2mn$ with $m, n \in \mathbb{N}$ and $m > n$ that there exist infinitely many Pythagorean triples. Write this result as theorem with proof in LaTeX. Further, add a table of the following form in which you list at least 5 Pythagorean triples.

a	b	c
3	4	5

Aufgabe 11.2. Write a `myenumerate`-environment with associated counter, which generates for a code

```
\begin{myenumerate}
  \myitem A
  \myitem B
  \myitem C
\end{myenumerate}
```

the following result

(*i*) A

(*ii*) B

(*iii*) C

where the numbering of the roman numbers is automatic. Build on the `itemize`-environment. Write therefore a macro `\myitem`, which uses the command `\item`. Check via the WWW how you could solve this exercise as well with the help of the `enumerate`-package.

Aufgabe 11.3. Inform yourself via the WWW about the `list`-environment. Write with the help of this environment, an environment `myitemize` such that

```
\begin{myitemize}
  \item A
  \item B
  \item C
\end{myitemize}
```

generates the following result

♠ A

♠ B

♠ C

The symbol ★ is generated via `$\spadesuit$`.

Aufgabe 11.4. Write a `theorem`- and a `lemma`-environment with the following layout. Here, $\square$ is generated via `\square`. For both environments the same counter should be used. The counter should depend on the chapter and the section. Optionally, one should be able to give the theorem resp. lemma a name. Use these environments in a document with at least one chapter (`chapter`) and two sections (`section`). Write in each section an arbitrary theorem and an arbirtrary lemma of your analysis lecture. Use always an appropriate `\label`.

Satz 1.1.2 (Bolzano-Weierstrass). In a finite dimensional normed space X, each bounded sequence $(x_n)_{n\in\mathbb{N}}$ has a convergent subsequence. $\square$

Lemma 1.1.3 (Zorn). *Suppose a partially ordered set P has the property that every chain has an upper bound in P. Then the set P contains at least one maximal element.* $\square$

Aufgabe 11.5. Write a `proof`-environment such that a proof is introduced via a bold-cursive ***Proof.***. The end of the proof (as part of the environment) should be indicated via a right-aligned $\blacksquare$ via `\blacksquare`. Formulate the following assertion as lemma, prove it with techniques of linear algebra and write the lemma with its proof in LaTeX, where all appearing references should be realized via `\label` and `\ref` etc. If $A \in \mathbb{R}^{n\times n}$ is a matrix with $\sum_{j,k=1}^n x_j A_{jk} x_k > 0$ fo all $x \in \mathbb{R}^n$, then A is regular.

Aufgabe 11.6. With the help of the previous exercises one can prove the *Lemma von Lax-Milgram* in the finite dimensional case: Let X be a finite dimensional vector space over $\mathbb{R}$ with the basis $\{v_1, \dots, v_n\}$, $F : X \to \mathbb{R}$ linear and $a(\cdot,\cdot) : X \times X \to \mathbb{R}$ a bilinear form on X, i.e. $a(\cdot,\cdot)$ is linear in both components. Further, we assume $a(v, v) > 0$ for all $v \in X$. Then there exists a unique $u \in X$ with $a(u, v) = F(v)$ for all $v \in X$. To prove this, one uses the approach $u = \sum_{k=1}^n x_k v_k$ and shows that the coefficient vector $x \in \mathbb{R}^n$ is unique. Formulate the Lemma of Lax-Milgram as theorem with proof in LaTeX and extend the document of the previous exercises. All appearing references should be realized via `\label` and `\ref` etc.

Aufgabe 11.7. Formulate the following result as theorem with proof in LaTeX and extend the document of the previous exercise. All appearing references should be realized via `\label` and `\ref` etc. Let X be a metric space. A sequence $(x_n)_{n\in\mathbb{N}} \subseteq X$ converges to some limit point $x \in X$, if each subsequence $(x_{n_j})_{j\in\mathbb{N}}$ contains a convergent subsequence $\left(x_{n_{j_k}}\right)_{k\in\mathbb{N}}$ which converges to x.

Aufgabe 11.8. Write an arbirtrary text with heading and at least 400 words and 10 proper names in LaTeX. Use 12pt as font size. Divide your text into at least 2 sections. Include the proper names into an index which is shown at the end of the document.