

## Übungen zur Vorlesung Computermathematik

### Serie 12

**Aufgabe 12.1.** Which journal hides behind the abbreviation *Appl. Numer. Math.*? What is the complete title? What is the correct abbreviation? Write bibliography which contains two articles of the last edition of this journal. Which journal has the abbreviation *Math. Comput.*? What is the complete title? What is the correct abbreviation? Extend your bibliography with two articles of the current edition of this journal. Further extend it with an English book from Wolfgang Hackbusch as well as his dissertation. To find the dissertation, you can use the *Mathematics Genealogy Project*, see <http://www.genealogy.ams.org>.

**Aufgabe 12.2.** Use <http://www.ams.org/mathscinet> to find the correct abbreviations of the journals *Annals of Mathematics*, *Archives of Computational Methods in Engineering* and *Inventiones mathematicae*. Which traffic light status do these journals have? Which editions are available for free? Write the result of your research in a 3-columned table of the form

title of the journal	abbreviation by <code>mathscinet</code>	availability
Numerische Mathematik	...	...

**Aufgabe 12.3.** Write a MATLAB-function `pythagoras` which calculates for given  $n \in \mathbb{N}$  and given file name `name`  $n$  different Pythagorean triples. The result should be written as a  $\LaTeX$  table in the file `name.tex`. Further, write a  $\LaTeX$ -document which includes such a table via `\input{name.tex}`.

**Aufgabe 12.4.** Plot in MATLAB the potential  $f(x, y) = x \cdot \exp(-x^2 - y^2)$  as graph in  $\mathbb{R}^2 \times \mathbb{R}$  as well as colored projection onto the plane, where we consider  $[-5, 5]^2 \subset \mathbb{R}^2$  as domain. Add a horizontal `colorbar` under the plots. Include the graphics in a  $\LaTeX$ -document. Use therefore the `figure`-environment with legend, where the pictures should be (via `minipage`) displayed one beside the other.

**Aufgabe 12.5.** For a differentiable function  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $x \in \mathbb{R}$  we define the **one-sided differential quotient**

$$\Phi(h) := \frac{f(x+h) - f(x)}{h} \quad \text{für } h > 0$$

and  $\Phi(0) := f'(x)$ . There holds  $\lim_{h \rightarrow 0} \Phi(h) = \Phi(0)$ . Prove with the help of the Taylor theorem that for  $f \in C^2(\mathbb{R})$  it holds

$$|\Phi(0) - \Phi(h)| = \mathcal{O}(h),$$

and determine the constant which hides in the Landau notation as accurate as possible. Write your result as a lemma with proof in  $\LaTeX$ .

**Aufgabe 12.6.** For  $f \in C^2(\mathbb{R})$  it holds  $e_h := |\Phi(h) - \Phi(0)| = \mathcal{O}(h)$  with the function  $\Phi$  of Aufgabe 12.5. For arbitrary  $f \in C^1(\mathbb{R})$ , one observes only  $e_h = \mathcal{O}(h^\alpha)$  for some  $\alpha \in (0, 1]$ . The constant  $\alpha$  is called **convergence order**. With the approach  $e_h = ch^\alpha$  the term  $\delta_h := |\Phi(h) - \Phi(h/2)|$  satisfies the estimate

$$e_h(1 - 2^{-\alpha}) \leq \delta_h \leq e_h(1 + 2^{-\alpha}),$$

i.e., it holds as well  $\delta_h = \mathcal{O}(h^\alpha)$ . With the further approach  $\delta_h = Ch^\alpha$  one obtains for  $h$  and  $h/2$  two equations from which one can calculate the **experimental convergence order** and the corresponding constant  $C$ :

$$\alpha = \log(\delta_h/\delta_{h/2})/\log(2) \quad \text{as well as} \quad C = \delta_h/h^\alpha.$$

Formulate the text of the exercise in your own words with all mathematical intermediate steps in L<sup>A</sup>T<sub>E</sub>X.

**Aufgabe 12.7.** Use the WWW to find out what the following L<sup>A</sup>T<sub>E</sub>X-code does. Prepare to being able to explain the code.

```

1 \newsavebox{\tmptext}%
2 \newenvironment{mycolorbox}[1]%
3 { \def\tmpcolor{#1}%
4   \begin{lrbox}{\tmptext}%
5     \begin{minipage}{\textwidth}%
6   }%
7 { \end{minipage}%
8   \end{lrbox}%
9   \colorbox{\tmpcolor}{\usebox{\tmptext}}}%
10 }
```

Write an example code in which you explain the application of the code.

**Aufgabe 12.8.** Write a L<sup>A</sup>T<sub>E</sub>X- file with the algorithm of the Gauss-elimination.

**Input:** Matrix  $A \in \mathbb{K}^{n \times n}$  mit LU-Zerlegung, rechte Seite  $b \in \mathbb{K}^n$

```

for k = 1, ..., n - 1
  for i = k + 1, ..., n
     $\ell_{ik} = a_{ik}^{(k)} / a_{kk}^{(k)}$ 
     $b_i^{(k+1)} = b_i^{(k)} - \ell_{ik} b_k^{(k)}$ 
    for j = k + 1, ..., n
       $a_{ij}^{(k+1)} = a_{ij}^{(k)} - \ell_{ik} a_{kj}^{(k)}$ 
    end
  end
end
```

**Output:** nicht-triviale Einträge der Matrizen  $L, U \in \mathbb{K}^{n \times n}$  mit  $u_{ij} := a_{ij}^{(i)}$  sowie modifizierte rechte Seite  $y \in \mathbb{K}^n$  mit  $y_i := b_i^{(i)}$ .

Write a C-fuction `void gauss(double** A, double* b, int n)` which realizes this procedure and which overwrites  $A$  and  $b$  appropriately. Include your C-code with the help of the `listing`-package in the same document.