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Übungen zur Vorlesung Computermathematik

Serie 10

Aufgabe 10.1. Write a LaTeX-file which consists of this exercise sheet– headline of the sheet up to and including Aufgabe 10.2. To generate a LaTeX-command `\command`, you can use `\verb|\command|`.

Aufgabe 10.2. Write a text of your choice with headline and at least 400 words in LaTeX. Divide your text in at least two sections and add a table of contents. Use 12pt as font size. What does the warning `Overfull hbox` mean? If necessary, modify the text such that LaTeX does not return this warning. Take a look at the generated log-file `text.log` and prepare to be able to explain the content of the file during your exercise. How would one have to modify the text in order to avoid the warning `Overfull hbox`? (Use the command `\-` resp. `\linebreak`. What is the difference between `\linebreak` and `\newline`?) Add the reference from where you took your text in a footnote `\footnote{...}`.

Aufgabe 10.3. Write the following definition of a Vandermonde matrix:

$$V := \begin{pmatrix} 1 & x_1 & x_2^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{pmatrix} \in \mathbb{R}^{n\times n}$$

in LaTeX. The dots are generated by `\cdots`, `\vdots` and `\ddots`, the symbol $\times$ by `\times`.

Aufgabe 10.4. Write a `myenumerate`-environment with associated counter, which generates for a code

```
\begin{myenumerate}
  \myitem A
  \myitem B
  \myitem C
\end{myenumerate}
```

the following result

(i) A

(ii) B

(iii) C

where the numbering of the roman numbers is automatic. Build on the `itemize`-environment. Write therefore a macro `\myitem`, which uses the command `\item`. Check via the WWW how you could solve this exercise as well with the help of the `enumerate`-package.

Aufgabe 10.5. Use `\newtheorem`, to generate a new *theorem*-environment. Write as well *proof*-environment. The proof should start (as part of the environment) with bold-italic ***Proof.*** The end of the proof (as part of the environment) should be indicated with a right-aligned `\blacksquare` $\blacksquare$, i.e., there is a right-aligned $\blacksquare$ at the end of the proof. Formulate and prove the following theorem in LaTeX.
Hint. Surely, you have seen the implication (ii) $\Rightarrow$ (i) in your Analysis-1-lecture. For the converse implication (i) $\Rightarrow$ (ii), recall that uniformly continuous functions map Cauchy-sequences onto Cauchy-sequences.

Theorem 1. *For $a, b \in \mathbb{R}$ and a continuous function $f : (a, b) \to \mathbb{R}$, the following two assertions are equivalent:*

(i) *f is uniformly continuous.*

(ii) *f has a continuous extension onto the compact interval $[a,b]$, i.e., there exists a function $\widehat{f}:[a,b]\to\mathbb{R}$ with $\widehat{f}(x)=f(x)$ for all $x\in(a,b)$.*

In this case the continuous extension $\widehat{f}$ is even unique.

Aufgabe 10.6. Formulate the following result as theorem with proof in LaTeX and extend the document of Aufgabe 10.5. All appearing references should be realised via `\label` and `\ref` etc. Write a suitable macro for sequences. A sequence $(x_n)_{n\in\mathbb{N}}$ of real numbers converges to some limit point $x\in\mathbb{R}$, if each subsequence $(x_{n_j})_{j\in\mathbb{N}}$ contains a convergent subsequence $\left(x_{n_{j_k}}\right)_{k\in\mathbb{N}}$ which converges to x.

Aufgabe 10.7. Formulate the following result as theorem with proof in LaTeX and extend the document of Aufgabe 10.5. All appearing references should be realised via `\label` and `\ref` etc.
Let $n\in\mathbb{N}$. It holds:

$$\sqrt{n}\in\begin{cases}\mathbb{N}, & \text{if } n \text{ is a square number,}\\ \mathbb{R}\setminus\mathbb{Q}, & \text{otherwise.}\end{cases}$$

Write a LaTeX-file which includes the assertion (formulated as theorem) and the (detailed) proof of this assertion. Use the environments from Aufgabe 10.5.
Hint. You may use the fact that each natural number x has a unique prime factorisation, i.e., there exists a unique finite sequence of prime numbers $2\le p_1\le\cdots\le p_k$ with $x=\prod_{j=1}^k p_j$.

Aufgabe 10.8. Formulate the following result as theorem with proof in LaTeX and extend the document of Aufgabe 10.5. All appearing references should be realised via `\label` and `\ref` etc. Write a suitable macro for norms and $\operatorname{dist}(\cdot,\cdot)$. Let $n\in\mathbb{N}$ and $A,B\subset\mathbb{R}^n$ open subsets with compact closure $\overline{A},\overline{B}$ and $A\cap B=\emptyset$. We define the boundary of the sets as $\partial A:=\overline{A}\setminus A$ and $\partial B:=\overline{B}\setminus B$ (the symbol ∂ is generated by `\partial`). Then, there holds for the distances of the two sets that $\operatorname{dist}(A,B)=\operatorname{dist}(\partial A,\partial B)$, where we define for arbitrary sets $C,D\subset\mathbb{R}^n$

$$\operatorname{dist}(C,D):=\inf\{\|x-y\|_2 : x\in C, y\in D\} \tag{1}$$

Hint. Show that $\operatorname{dist}(A,B)=\operatorname{dist}(\overline{A},\overline{B})$. Next, note that the infimum in (1) is a minimum for compact sets C,D.