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Übungen zur Vorlesung
Computermathematik

Serie 11

Aufgabe 11.1. Write a LaTeX-file in which the following theorem of Brezzi is formulated. Define suitable macros for the norms as well as the bilinear forms $a(\cdot,\cdot)$ und $b(\cdot,\cdot)$.

Theorem (Brezzi 1974). Let X and Y be hilbert spaces. Further, let $a : X \times X \to \mathbb{R}$ and $b : X \times Y \to \mathbb{R}$ be continuous bilinear forms and $X_0 := \big\{x \in X \,:\, b(x,\cdot) = 0 \in Y^*\big\}$. Under the assumptions

- $\alpha := \inf\limits_{v \in X_0 \backslash \{0\}} \dfrac{a(v,v)}{\|v\|_X^2} > 0$, i.e., $a(\cdot,\cdot)$ is elliptic auf X_0,
- $\beta := \inf\limits_{y \in Y \backslash \{0\}} \sup\limits_{x \in X \backslash \{0\}} \dfrac{b(x,y)}{\|x\|_X \|y\|_Y} > 0$.

there holds the assertion: For each $(x^*, y^*) \in X^* \times Y^*$ there is a unique solution $(x,y) \in X \times Y$ of the so-called saddle point problem

$$\begin{array}{lcll} a(x,\widetilde{x}) \;+\; b(\widetilde{x},y) & = & x^*(\widetilde{x}) & \text{for all } \widetilde{x} \in X, \\ b(x,\widetilde{y}) & = & y^*(\widetilde{y}) & \text{for all } \widetilde{y} \in Y. \end{array} \tag{1}$$

Aufgabe 11.2. Which journal hides behind the abbreviation *Math. Mod. Meth. Appl. S.*? What is the complete title? What is the correct abbreviation? Write bibliography which contains two articles of the last edition of this journal. Which journal has the abbreviation *Math. Comput.*? What is the complete title? What is the correct abbreviation? Extend your bibliography with two articles of the current edition of this journal. Further extend it with an English book from Stefan Sauter as well as his dissertation. To find the dissertation, you can use the *Mathematics Genealogy Project*, see `http://www.genealogy.ams.org`.

Aufgabe 11.3. Write an arbirtrary text with heading and at least 400 words and 10 proper names in LaTeX. Use 12pt as font size. Divide your text into at least 2 sections. Include the proper names into an index which is shown at the end of the document.

Aufgabe 11.4. Three natural numbers $a, b, c \in \mathbb{N}$ are called *Pythagorean triple*, if $a^2 + b^2 = c^2$. Prove via the approach $a := m^2 - n^2$ and $b := 2mn$ with $m, n \in \mathbb{N}$ and $m > n$ that there exist infinitely many Pythagorean triples. Write this result as theorem with proof in LaTeX. All appearing references should be realised via `\label` and `\ref` etc. Further, add a table of the following form in which you list at least 5 Pythagorean triples.

a	b	c
3	4	5

Aufgabe 11.5. Let $f \in C^2(\mathbb{R})$. For $I = [a,b]$ and $N \in \mathbb{N}$ let $h := (b-a)/N$. We set $x_i := a + ih$, $i = 0, \dots, N$ and define the *rectangle rule*

$$R(h) := \sum_{i=0}^{N-1} h f(x_i)$$

as an approximation for $\int_a^b f(x)\,dx$. Show that the error satisfies $|\int_a^b f(x)\,dx - R(h)| = \mathcal{O}(h)$. Determine therefore a number $C > 0$ for which you can prove

$$\Big| \int_a^b f(x)\,dx - R(h) \Big| \le C\,h \max_{x \in [a,b]} |f'(x)|.$$

Write a LaTeX-file which includes the assertion and the (detailed) proof of the product rule for the n-th derivative. All appearing references should be realised via `\label` and `\ref` etc. Use a macro of the form $\int_{(\cdot)}^{(\cdot)} (\cdot)\, d(\cdot)$.
Hint. Consider the intervals $[x_i, x_{i+1}]$ and use Taylor.

Aufgabe 11.6. Visualise the theoretical results from Aufgabe 11.5. To this end, make a double-logarithmic convergence plot in MATLAB for two different functions f_1, f_2. Label the axes suitably. Export the pictures as `eps`-files by `print` (see MATLAB-slide 106). Include the graphics in a LaTeX-document. Use a `figure`-environment with legend, where the pictures should be placed (by `minipage`) side by side. Replace the labels of the axes by `\psfrag`.

Aufgabe 11.7. Let I be a nonempty open interval. Then it holds for $f, g \in C^\infty(I)$ and $n \in \mathbb{N}$

$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}.$$

Write a LaTeX-file which includes the assertion and the (detailed) proof of the product rule for the n-th derivative. All appearing references should be realised via `\label` and `\ref` etc.

Aufgabe 11.8. Write a sort algorithm of your choice in MATLAB (you must not use the command `sort`). Copy your code in a suitable LaTeX-environment. Compare your algorithm with the MATLAB-command `sort`. Therefore, generate 5 random vectors of length 10^j, $j = 4, \ldots, 8$, and consider the required computational times. Write your results in a LaTeX-tabular of the following form.

N	10^4	10^5	10^6	10^7	10^8
`sort`					
`mysort`					