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Übungen zur Vorlesung Computermathematik

Serie 10

Aufgabe 10.1. Write a LaTeX-file which consists of this exercise sheet– headline of the sheet up to and including Aufgabe 10.2. To generate a LaTeX-command `\command`, you can use `\verb|\command|`.

Aufgabe 10.2. Write a text of your choice with headline and at least 400 words in LaTeX. Divide your text in at least two sections and add a table of contents. Use 12pt as font size. What does the warning `Overfull hbox` mean? If necessary, modify the text such that LaTeX returns this warning. Take a look at the generated log-file `text.log` and prepare to be able to explain the content of the file during your exercise. How would one have to modify the text in order to avoid the warning `Overfull hbox`? (Use the command `\-` resp. `\linebreak`. What is the difference between `\linebreak` and `\newline`?) Add the reference from where you took your text in a footnote `\footnote{...}`.

Aufgabe 10.3. Write the following definition of the characteristic polynomial of a matrix $A \in \mathbb{R}^{n\times n}$

$$p(t) = \det(A - t\cdot \mathrm{Id}) = \begin{vmatrix} a_{11}-t & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-t & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-t \end{vmatrix}$$

in LaTeX. Note the symbol Id instead of Id for the identity matrix. **Hint.** To generate the matrix, see slide 55.

Aufgabe 10.4. Write the following text in LaTeX, where the symbol $\pm$ is generated by `\pm`: For given ***basis*** $b \in \mathbb{N}_{\ge 2}$, ***mantissa length*** $t \in \mathbb{N}$ and ***exponential bounds*** $e_{\min} < 0 < e_{\max}$ we define the set of ***normalized floating point numbers*** $\mathbb{F} := \mathbb{F}(b, t, e_{\min}, e_{\max}) \subset \mathbb{R}$ by

$$\mathbb{F} = \{0\} \cup \Big\{ \Big(\sigma \sum_{k=1}^{t} a_k b^{-k}\Big) b^e \,\Big|\, \sigma \in \{\pm 1\}, a_j \in \{0, \dots, b-1\}, a_1 \neq 0, e \in \mathbb{Z}, e_{\min} \le e \le e_{\max} \Big\}.$$

The finite sum $a = \sum_{k=1}^{t} a_k b^{-k}$ is called ***normalized mantissa*** of a floating point number.

Aufgabe 10.5. Write the following text in LaTeX: The Gamma function is defined as

$$\Gamma(x) := \lim_{n\to\infty} \frac{n!\,n^x}{x(x+1)\cdots(x+n)}.$$

There holds the Weierstraß product representation

$$\frac{1}{\Gamma(x)} = x \cdot e^{Cx} \cdot \prod_{k=1}^{\infty} \Big(1 + \frac{x}{k}\Big) e^{-x/k} \quad \text{mit} \quad C := \lim_{n\to\infty} \Big(\sum_{k=1}^{n} \frac{1}{k} - \ln n \Big).$$

Here, `\infty` is the symbol ∞, and $\cdot$ resp. $\cdots$ is obtained by `\cdot` resp. `\cdots`.

Aufgabe 10.6. Write the following formula in LaTeX-file: For $q \in \mathbb{R}$, it holds that

$$\lim_{n\to\infty} q^n = \begin{cases} +\infty & \text{falls } q > 1, \\ 1 & \text{falls } q = 1, \\ 0 & \text{falls } -1 < q < 1, \\ \nexists & \text{falls } q \le -1. \end{cases}$$

The symbol $\nexists$ is generated via `\nexists` or `\not\exists`. **Hint.** See slide 55 to realize the cases.

Aufgabe 10.7. Write the following text in LaTeX: Let $\Omega \subseteq \mathbb{R}^d$ (with $d \geq 3$) be a bounded domain with Lipschitz-boundary and $u \in C^2(\Omega)$ a solution of the Laplace equation $\Delta u := \sum_{i=1}^d \frac{\partial}{\partial x_i} u = 0$. Then, there holds the representation formula

$$\forall x \in \Omega : \quad u(x) = \frac{1}{4\pi} \int_{\partial\Omega} \frac{1}{|x-y|} \frac{\partial}{\partial \nu(y)} u(y)\, dy - \frac{1}{4\pi} \int_{\partial\Omega} \Big(\frac{\partial}{\partial \nu(y)} \frac{1}{|x-y|} \Big) u(y)\, dy.$$

Here, `\partial` is the symbol ∂.

Aufgabe 10.8. Formally a triangle T with vertices $x, y, z \in \mathbb{R}^2$ is defined as convex hull of these points

$$\mathrm{conv}(x,y,z) := \{ax + by + cz : \ a, b, c \geq 0 \text{ with } a + b + c = 1\}.$$

The triangle T is called non-degenerated if the vectors $y - x$ and $z - x$ are linearly independent. Formulate the following result with proof in LaTeX. Let $T = \mathrm{conv}(x,y,z)$ and $\widetilde{T} = (\widetilde{x}, \widetilde{y}, \widetilde{z})$ be two non-degenerated triangles. Then, there exists an affine bijection $\Phi : T \to \widetilde{T}$, i.e., a bijective mapping of the form $\Phi(v) = Av + b$ with a matrix $A \in \mathbb{R}^{2\times 2}$ and a vector $b \in \mathbb{R}^2$. Here, the symbol $\widetilde{T}$ is obtained by `\widetilde T`. The symbol $\times$ is obtained by `\times`. Note the symbol conv instead of *conv* for the convex hull.