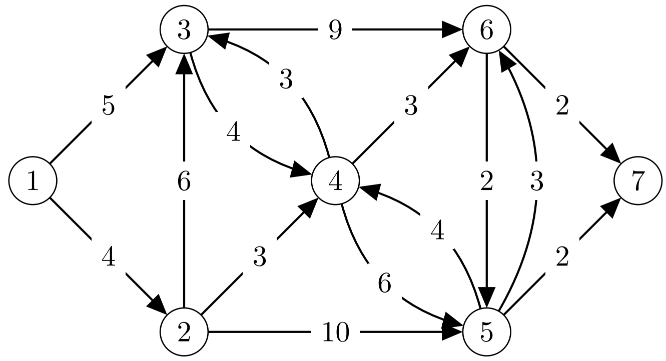
3. ÜBUNG
104.283 Diskrete Mathematik für Informatik

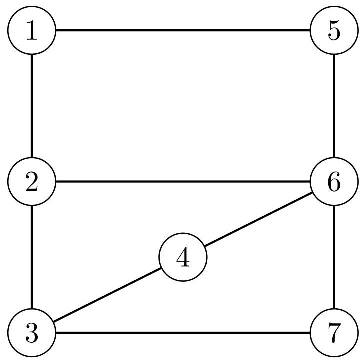
(31) Compute the minimal distances between all pairs of vertices in the following graph using the Floyd-Warshall algorithm:



(32) (a) The line graph $\bar{G}$ of a simple undirected graph $G = (V, E)$ is the (simple) graph with vertex set E, with an edge between two vertices of $\bar{G}$, if and only if the corresponding edges are incident to a common vertex of G. Show that the line graph of an Eulerian graph is Eulerian and Hamiltonian, and that the line graph of a Hamiltonian graph is also Hamiltonian. If $\bar{G}$ is Hamiltonian, can we conclude that G is Hamiltonian?

(b) Is a subdivision of an Eulerian graph Eulerian? Is a subdivision of a Hamiltonian graph Hamiltonian?

(33) Compute the closure of the following graph:



(34) Show that the n-dimensional hypercube is Hamiltonian for $n \geq 2$.

(35) For which m and n does the complete bipartite graph $K_{m,n}$ have a Hamiltonian cycle?

For the following exercises, use a suitable graph model to reformulate the exercises as graph theoretical problems.

(36) Given a subset $A \subseteq \mathbb{R}^2$ with area a and two decomposition of A into subsets $A_1, A_2, \ldots, A_m$ and $B_1, B_2, \ldots, B_m$ such that all the sets A_i and B_i have the same area a/m. Prove that there exists a permutation π of $\{1, 2, \ldots, m\}$ such that for all $i = 1, \ldots, m$ we have $A_i \cap B_{\pi(i)} \neq \emptyset$.

(37) Given a set A with n elements and $B = \{A_1, A_2, \ldots, A_n\} \subseteq 2^A$. Prove that there exists an injective mapping $f : B \to A$ such that $f(A_i) \in A_i$ for all $i \in \{1, 2, \ldots, n\}$ if and only if for all $I \subseteq \{1, 2, \ldots, n\}$ the cardinality of $\bigcup_{i \in I} A_i$ is at least equal to the cardinality of I.

(38) Follow the hint below to construct a schedule for the matches in a league of 2n teams which meets the following constraints:

- In each round each team plays exactly one match.

- In the end each team must have played against each of the other teams exactly once.

Hint: Consider the graph K_{2n} on the vertex set $\{1, 2, \dots, 2n\}$ and show that each of the sets $M_i = \{1i\} \cup \{xy : x + y \equiv 2i \mod 2n - 1 \text{ and } x \neq y, x \neq 1, y \neq 1\}$ is a perfect matching for $i \in \{2, \dots, 2n\}$.

(39) Let M be a matching of a simple, undirected graph $G = (V, E)$. A path P in G is *alternating* if exactly every second edge of P is in M. An alternating path *extends* if the both the first and the last vertices of P are not covered by an edge in M. Prove for an extending alternating path P that $M \triangle P := (M\ W) \cup (W\ M)$ is a matching and $|M \triangle W| = |M| + 1$.

(40) Let G be an undirected simple graph and H a subgraph of G satisfying $H \cong K_n$ for some n. What can be said about the relation between $\chi(G)$ and $\chi(H)$? Find a graph G with $\chi(G) = 3$ which does not have a K_3 as a subgraph.