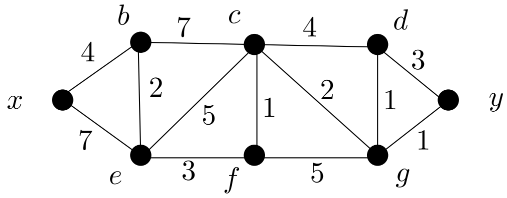
UE Discrete Mathematics

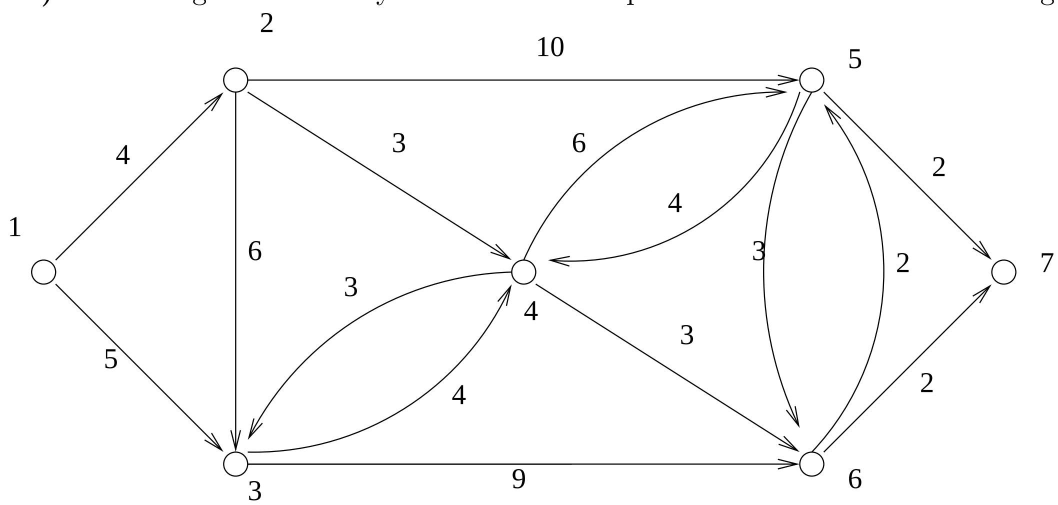
Exercises for Nov 6/7, 2013

21) Use Dijkstra's algorithm to determine $d(x, y)$ in the following graph.



22) Use one of the algorithms presented in the lecture to construct a spanning tree which contains all the shortest paths connecting vertex x with all the other vertices in the graph of Exercise 21.

23) Use the algorithm of Floyd-Warshall to compute all distances in the following graph.



24) Find a graph $G = (V, E)$ and two vertices $x, y \in V$ such that Dijkstra's algorithm does not compute the distance $d(x, y)$ correctly.

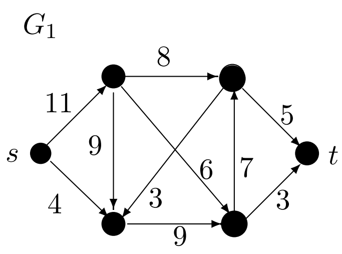
25) The matrix W corresponds to the weight function w of flow network $G = (V, E, w, s, t)$ and the matrix $\mathbf{\Phi}$ to a flow ϕ on G.

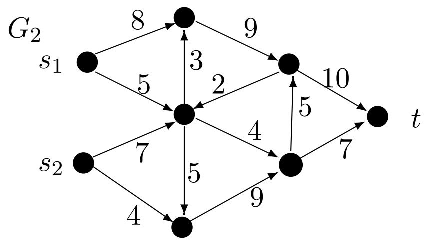
$$W = \begin{pmatrix} 0 & 5 & 7 & 8 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 5 & 3 & 11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{\Phi} = \begin{pmatrix} 0 & 5 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Determine $v(\phi)$.

(b) Find an augmenting path constsing of forward edges only and an augmenting path with at least one backward edge.

(c) Find a minimal cut.





(d) Find a maximal flow on G.

26) Use the algorithm of Ford and Fulkerson to compute a maximal flow in the network G_1 below!

27) Use the algorithm of Ford and Fulkerson to compute a maximal flow in the network G_2 which has two sources s_1 and s_2!

28) For a simple and undirected graph G we define the *line graph* $\bar{G}$ as follows: $V(\bar{G}) = E(G)$ and $(e, f) \in E(\bar{G})$ if and only if the edges e and f share a vertex. Prove that the line graph of an Eulerian graph is Eulerian and Hamiltonian!

29) Let G_n denote the n-dimensional hypercube. Show that G_n is Hamiltonian if $n \geq 2$.

30) Prove that a graph G is bipartite if and only if each cycle in G has even length.