# UE Discrete Mathematics

# Exercises for Dec 4/5, 2013

**61)** Solve the following recurrence using generating functions: $a_{n+1} = a_n + (n+1)^2$, $a_0 = 1$.

**62)** Solve the following recurrence using generating functions: $a_{n+2} = 3a_{n+1} - 2a_n$, $a_0 = 1$, $a_1 = 3$.

**63)** Use generating functions to find a closed form expressions for the sum $\sum\limits_{k=0}^{n} (k^2 + 3k + 2)$.

**64)** Compute

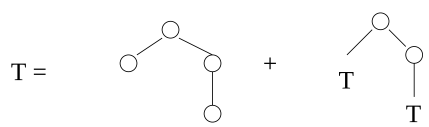
$$[z^n] \frac{2 + 3z^2}{\sqrt{1 - 5z}}.$$

**65)** Prove the following identity:

$$\sum_{n \geq 0} \binom{2n}{n} z^n = \frac{1}{\sqrt{1 - 4z}}.$$

**66)** A $t$-ary tree is a plane rooted tree such that every node has either $t$ or 0 succesors. A node with $t$ succesors is called internal nodes. How many leaves has a $t$-ary tree with $n$ internal nodes? Moreover, let $a_n$ be the number of $t$-ary trees with $n$ internal nodes and $A(z)$ the generating function of this sequence. Find a functional equation for $A(z)$!

**67)** Compute the numbers $t_n$ of plane rooted trees with $n$ nodes which can be described by the equation




**68)** Compute the number of plane rooted trees with $n$ nodes.

**69)** Consider a regular $(n+2)$-gon $A$, say, with the vertices $0, 1 \ldots, n+1$. A triangulation is a decomposition of $A$ into $n$ triangles such that the 3 vertices of each triangle are vertices of $A$ as well. Show that the set $\mathcal{T}$ of triangulations of regular polygons can be described as a combinatorial construction satisfying

$$\mathcal{T} = \{\varepsilon\} \cup \mathcal{T} \times \Delta \times \mathcal{T}$$

where $\Delta$ denotes a single triangle and $\varepsilon$ denotes the empty triangulation (consisting of no triangle and corresponding to the case $n = 0$). What is the number of triangulations of $A$?

**70)** Let $\mathcal{L}$ denote the set of words over the alphabet $\{a, b\}$ that contain exactly $k$ occurrences of $b$. Obviously, the number of words in $\mathcal{L}$ which have exactly $n$ letters is $\binom{n}{k}$. Prove this by finding a specification of $\mathcal{L}$ as combinatorial construction and translating this specification into generating functions.