Übungstest – Discrete Mathematics - WS 2014 (Gruppe Probetest)

1. (a) State the handshaking lemma for graphs.
   (b) Show that a tree without vertices of degree 2 has more leaves than internal nodes.

---

2. (a) Let (E, I) be a matroid, and A and B are in I with $|B| = |A| + 1$. What do we then know about $B \setminus A$?
   (b) Let $G = (V_1 \cup V_2, E)$ be a bipartite graph. Let

$$I = \{A \subseteq V_1 : \text{there is a matching of } G \text{ that covers the vertices in } A\}.$$

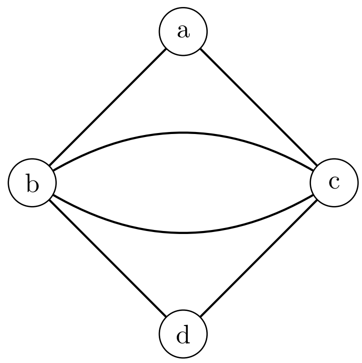
   We know that (V_1, I) is a matroid. Show that, however, the pair (E, J) with

$$J = \{M \subseteq E : M \text{ is a matching of } G\}$$

   is, in general, not a matroid. Hint: it is sufficient to provide very small counterexample, eg. with four vertices and three edges.

---

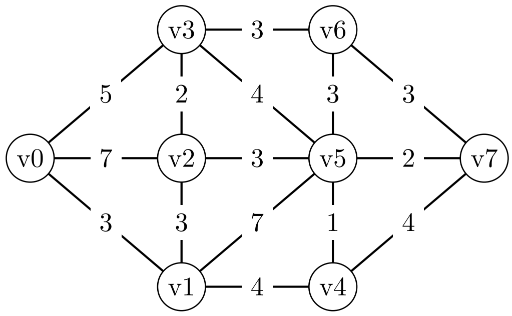
3. Let G be the graph



   (a) What is the adjacency matrix of G?
   (b) Compute the number of spanning trees of the graph using the matrix tree theorem.
   (c) Compute the number of walks of length two between all pairs of vertices of G.

---

4. Let G be the following graph:



   (a) Use Dijkstra's algorithm to compute a distance tree from v_0 to the other vertices in G.
   (b) Use Kruskal's algorithm to compute a minimum spanning tree of G.
