Quiz 1

1. Verwenden Sie das Prinzip der vollständigen Induktion um

$$\sum_{k=1}^{n} \frac{6}{(k+3)(k+2)} = \frac{2n}{n+3}$$

für $n \in \mathbb{N}$ zu zeigen.

$\underline{n=1:}$ $\frac{6}{4\cdot 3} = \frac{1}{2} \overset{\checkmark}{=} \frac{1}{2} = \frac{2}{4} = \frac{2n}{n+3}$.

$\underline{n \Rightarrow n+1:}$ $\sum_{k=1}^{n+1} \frac{6}{(k+3)(k+2)} = \sum_{k=1}^{n} \frac{6}{(k+3)(k+2)} + \frac{6}{(n+4)(n+3)} =$

$= \frac{2n}{n+3} + \frac{6}{(n+4)(n+3)} = \frac{2n(n+4)+6}{(n+4)(n+3)} = \frac{2n^2+8n+6}{(n+4)(n+3)} = \frac{2(n^2+4n+3)}{(n+4)(n+3)}$ $\Leftrightarrow$

S.C.: $n = \frac{-4 \pm \sqrt{16-12}}{2} = -2 \pm 1 \leadsto n_1 = -1, \quad n_2 = -3$

$\Leftrightarrow$ $\frac{2(n+1)\cancel{(n+3)}}{(n+4)\cancel{(n+3)}} = \frac{2(n+1)}{(n+1)+3}$ ✓

ODER $\longrightarrow$

2. Zeigen Sie dass $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

$\sqrt[n]{n} = e^{\log n^{\frac{1}{n}}} = e^{\frac{1}{n}\log n} = e^{\frac{\log n}{n}}$

$\lim_{n\to\infty} \underset{\frac{\infty}{\infty}}{\frac{\log n}{n}} \overset{\text{L'H.R.}}{=\!=} \lim_{n\to\infty} \frac{1}{n} = 0$

$\lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} e^{\frac{\log n}{n}} = e^{\lim_{n\to\infty} \frac{\log n}{n}} = e^0 = 1$.

Induktionsanfang: $ls = \sum_{k=1}^{1} \frac{6}{(k+3)(k+2)} = \frac{6}{4 \cdot 3} = \frac{1}{2}$.
$(n=1)$

$rs = \frac{2}{1+3} = \frac{1}{2}$. ✓

Induktionsannahme: $\sum_{k=1}^{n} \frac{6}{(k+3)(k+2)} = \frac{2n}{n+3}$
(n)

Induktionsbehauptung: $\sum_{k=1}^{n+1} \frac{6}{(k+3)(k+2)} = \frac{2(n+1)}{(n+1)+3}$
$(n+1)$

Induktionsschritt:
$(n \Rightarrow n+1)$

$$ls = \sum_{k=1}^{n+1} \frac{6}{(k+3)(k+2)} = \sum_{k=1}^{n} \frac{6}{(k+3)(k+2)} + \frac{6}{(n+4)(n+3)} = \frac{2n}{(n+3)} + \frac{6}{(n+4)(n+3)}$$

$$= \frac{(2n)(n+4)+6}{(n+3)(n+4)} = \frac{2n^2+8n+6}{(n+4)(n+3)}$$

$$rs = \frac{2(n+1)}{(n+4)} \frac{(n+3)}{(n+3)} = \frac{(2n+2)(n+3)}{(n+4)(n+3)} = \frac{2n^2+2n+6n+6}{(n+4)(n+3)} = \frac{2n^2+8n+6}{(n+4)(n+3)}$$

$ls = rs$. ✓