

Differential geometry (104.358)

Exercise sheet for 14.6.2018

44. Let X be a surface and suppose that along a curve $t \mapsto C(t) = X(u(t), v(t))$ the surface is tangent to a fixed plane, i.e., the tangent planes of X along C are all the same.

Show that the points of the curve C are parabolic or flat points of X , and thus the Gauss curvature of X vanishes at these points.

45. Find all geodesics on a unit sphere with given initial point and velocity.

Hint: Do not parametrise the sphere.

46. Let X_1 and X_2 be two surfaces that intersect along a curve C . Suppose that the Gauss maps of the two surfaces are linearly independent along C .

Show that C is a pre-geodesic line of both X_1 and X_2 if and only if C is a line segment.

47. Prove that $K = -\frac{(\sqrt{G})_{rr}}{\sqrt{G}}$ in geodesic polar coordinates (r, θ) .

48. Compute the geodesic equations in geodesic polar coordinates (r, θ) .