Differential geometry (104.358)
Exercise sheet for 21.6.2018

49. Use the implicit mapping theorem to show directly that an implicitly defined manifold has local parametrisations.

50. Let $F_1, F_2 : \mathcal{E}^3 \ni U \to \mathbb{R}$ with $(\operatorname{grad} F_1 \times \operatorname{grad} F_2)(p) \neq 0$ for all $p \in U$. Prove that the equations
$$F_1(p) = F_2(p) = 0$$
defines a 1-dimensional manifold in $\mathcal{E}^3$. Hence, show that the conic sections
$$C_\alpha = \{O + e_1 x + e_2 y + e_3 z \,|\, x^2 + y^2 = z^2,\, x \cos\alpha + z \sin\alpha = d\},$$
where $\alpha \in \mathbb{R}$ and $d \neq 0$, are 1-dimensional submanifolds of $\mathcal{E}^3$.

51. Show that the graph $\{O + e_1 x + e_2 y + e_3 f(x, y)\}$ of a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ is a 2-dimensional submanifold of $\mathcal{E}^3$.

52. Suppose that $U \subset \mathcal{E}^n$ is an open set. Show that U is a submanifold of $\mathcal{E}^n$ and compute its tangent space.

53. Show that the general linear group $Gl(n) = \{A \in \operatorname{End}(\mathbb{R}^n) : \det(A) \neq 0\}$ is an n^2-dimensional submanifold of $\operatorname{End}(\mathbb{R}^n)$.

    Show that the special linear group $Sl(n) = \{A \in \operatorname{End}(\mathbb{R}^n) : \det(A) = 1\}$ is a $(n^2 - 1)$-dimensional submanifold of $\operatorname{End}(\mathbb{R}^n)$.