Differential geometry (104.358)
Exercise sheet for 24.5.2018

33. Show that all the points on the sphere with radius $r > 0$ are umbilic points and calculate its Gauss and mean curvature.

    Hint: Don't use an explicit parametrisation of the sphere.

34. Compute the Christoffel symbols of a conformally parametrised surface.

35. Let Σ denote the matrix representation of the shape operator. Prove that the partial covariant derivatives, $\nabla_{\frac{\partial}{\partial u}} S$ and $\nabla_{\frac{\partial}{\partial v}} S$ have matrix representations

$$\Sigma_u + [\Gamma_1, \Sigma] \quad \text{and} \quad \Sigma_v + [\Gamma_2, \Sigma],$$

    where $[.,.]$ denotes the usual commutator of matrices. Derive an expression of the Codazzi equation in terms of these.

36. Let X be a surface whose image lies on a sphere with radius $r > 0$. Show that

$$RY = \frac{1}{r^2} Y \times X_u \times X_v$$

    by using the identity

$$RY = \nabla_{\frac{\partial}{\partial u}} \nabla_{\frac{\partial}{\partial v}} Y - \nabla_{\frac{\partial}{\partial v}} \nabla_{\frac{\partial}{\partial u}} Y.$$

37. Prove that for a conformally parametrised surface the Gauss equation reads

$$K = -\frac{1}{2E} \Delta \ln E.$$