Differential Geometry (104.358)
Exercise sheet for 23.5.2019

36. Show that if a minimal surface ($H \equiv 0$) admits an isometric parametrisation then it is part of a plane.

37. Suppose that a surface $O + e_1x + e_2y + e_3z$ is implicitly defined through $F(x, y, z) = 0$ where $F : \mathbb{R}^3 \to \mathbb{R}$ is a smooth function satisfying $\operatorname{grad} F \neq 0$ wherever $F(x, y, z) = 0$. A parametrisation $X : (u, v) \mapsto X(u, v)$ of this surface therefore satisfies $F \circ (X - O) = 0$. Let $C : t \mapsto C(t)$ be a curve on this surface, i.e., $F \circ (C - O) = 0$.

    Show that the natural ribbon of C is given by (C, N) where

    $$N = \pm \frac{\operatorname{grad} F \circ (C - O)}{|\operatorname{grad} F \circ (C - O)|}.$$

    Use this to prove that the two curves

    $$C_\pm : t \mapsto C_\pm(t) = O + e_1 + e_2t \pm e_3t$$

    on the one-sheeted Hyperboloid

    $$\{O + e_1x + e_2y + e_3z \in \mathcal{E}^3 \mid x^2 + y^2 - z^2 - 1 = 0\}$$

    are asymptotic and pre-geodesic lines, but not curvature lines.

38. Prove that for a curvature line parametrisation, the Codazzi equations have the form

    $$0 = \kappa_v^+ + \frac{E_v}{2E}(\kappa^+ - \kappa^-) = \kappa_u^- - \frac{G_u}{2G}(\kappa^+ - \kappa^-).$$

39. Find a curvature line reparametrisation of the helicoid:

    $$X(u, v) = O + e_1 \sinh u \cos v + e_2 \sinh u \sin v + e_3v.$$