Differential Geometry (104.358)
Exercise sheet for 27.6.2019

44. Using the geodesic equations, find all geodesics on the circular cylinder

$$X(u, v) = O + e_1 \cos u + e_2 \sin u + e_3 v$$

with given initial point and velocity.

45. Let

$$X(u, v) = O + e_1 r(u) \cos v + e_2 r(u) \sin v + e_3 h(u)$$

be a surface of revolution. Prove that:

a) if a circle of latitude $t \mapsto X(u, t)$ is geodesic, then $r'(u) \equiv 0$;

b) if $r'^2 + h'^2 \equiv 1$, then the profile curves $t \mapsto X(t, v)$ are geodesic.

46. Compute the geodesic equations in geodesic polar coordinates (r, θ).

47. Prove that $K = -\frac{(\sqrt{G})_{rr}}{\sqrt{G}}$ in geodesic polar coordinates (r, θ).

Question to be handed in (written neatly or typed) on 27/06/19:

a) Without using an explicit parametrisation, find all geodesics on a sphere with given initial point and velocity.

b) Consider the parametrisation of the unit sphere

$$X(u, v) = O + e_1 \cos u \cos v + e_2 \cos u \sin v + e_3 \sin u.$$

Compute the geodesic equations of X. (You don't need to solve these equations!)

c) Show that if a geodesic on a surface lies in a fixed plane, then it must either be part of a straight line or a curvature line of the surface.

d) Show that if all of the geodesics on a surface are plane curves, then the surface must be part of a plane or sphere.