

Differential Geometry (104.358)

Exercise sheet for 28.3.2019

9. Let (X, N) be a ribbon for the curve $X : \mathbb{R} \supseteq \mathcal{I} \rightarrow \mathcal{E}^3$ and let $\tilde{X} := X \circ \psi$ be a reparametrisation of X , i.e., $\psi : \tilde{I} \rightarrow I$ is a bijective differentiable function with $\psi' \neq 0$.

(a) Show that (\tilde{X}, \tilde{N}) with $\tilde{N} = N \circ \psi$ is a ribbon.

(b) How does the curvatures κ_n , κ_g and torsion τ change between (X, N) and (\tilde{X}, \tilde{N}) ?

10. Let (X, N) be a ribbon for the curve $X : \mathbb{R} \supseteq I \rightarrow \mathcal{E}^3$ and let $\tilde{N} := N \cos \phi + B \sin \phi$, where $\phi : I \rightarrow \mathbb{R}$ is differentiable and $B = T \times N$.

Show that (X, \tilde{N}) is a ribbon and calculate how the curvatures κ_n , κ_g and the torsion τ change when switching from (X, N) to (X, \tilde{N}) .

11. Consider the parametrised curve

$$X : \mathbb{R} \rightarrow \mathcal{E}^3,$$

$$t \mapsto O + e_1 t + e_2 t^2 + e_3 \frac{2t^3}{3}.$$

Calculate the arc length of X .

Find a normal field N for X and calculate at $t = 0$ the normal curvature, geodesic curvature and torsion by using the identities

$$\kappa_n = -\frac{(N', T)}{|X'|}, \quad \kappa_g = -\frac{(T', B)}{|X'|}, \quad \tau = \frac{(N', B)}{|X'|}.$$

12. Let $X : \mathbb{R} \supseteq I \rightarrow \mathcal{E}^3$ be a regular parametrisation of a line, i.e., $X' \times X'' = 0$, and let F be an adapted frame for X .

Show that $\kappa_n = 0 = \kappa_g$ and find a unit normal field N so that $\tau = 1$.

13. Prove:

An arc length parametrised curve X in \mathcal{E}^3 lies in a plane if and only if there exists an adapted frame such that $\kappa_g = 0 = \tau$.

Is the same statement true for $\kappa_n = 0 = \tau$ and for $\kappa_n = 0 = \kappa_g$?