Problem 6. A surface of revolution is given by

$$\mathbf{x}(u,v) = \begin{pmatrix} \cos v & -\sin v & 0 \\ \sin v & \cos v & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x(u) \\ 0 \\ z(u) \end{pmatrix}$$

- Let $\big((x(u), z(u)\big) = (r\cos u + a, r\sin u)$, $a > 0, r > 0$. Display $\mathbf{x}$ over $[0, 2\pi] \times [0, 2\pi]$ in MATLAB.
- Write a function that computes the Gaussian as well as the mean curvature and display those values color-coded across the surface. Use a color gradient from blue (negative) over green (zero) to red (positive).

Problem 7. Let the points $\mathbf{p}_i \in \mathbb{R}^3$, $i = 1, \ldots, n$, be given as the rows of the matrix $P \in \mathbb{R}^{n\times 3}$. We want to estimate principal curvatures κ_1, κ_2 and corresponding curvature directions.

- For each point $\mathbf{p}_i$ find a suitable neighborhood $\mathbf{p}_1, \ldots, \mathbf{p}_m$ and define a local coordinate system using PCA.
- Use the jet-fitting procedure to estimate principal curvature directions, principal curvatures and the Gaussian curvature.
- Display the resulting curvature directions as small arrows/lines and color the surface according to Gaussian curvature as in Problem 6.

Data sets can be downloaded from TISS.

Problem 8. Prove Euler's formula.

Problem 9. Robust circle and line fitting: Compare the result of circle/line fitting using least squares, weighted least squares, and the RANSAC procedure.

Problems due November 9, 2017.