Problem 10. Given an ordered set $\{\mathbf{x}_i\}$, $i = 1, \ldots, n$, of points in $\mathbb{R}^d$, $d = 2, 3$, implement an algorithm to fit a Bézier curve $\mathbf{c}$ of degree 3:

- Data can be downloaded from TISS. $\{\mathbf{x}_i\}$ is stored as a matrix $X \in \mathbb{R}^{n \times d}$.
- Use chord length parameters (scale the obtained sequence t_i to $[0, 1]$).
- Look up the Matlab functions `spmak`, `spval`, and `spcol`. An example of how to use those functions is provided on TISS.
- Compute $\mathbf{c}$ using the above functions.

Problem 11. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function. Consider the surface

$$\mathbf{x} : \mathbb{R}^2 \to \mathbb{R}^3$$
$$\begin{pmatrix} u \\ v \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \\ f(u, v) \end{pmatrix}$$

and the problem of projecting a point $\mathbf{p} = (x, y, z)$ orthogonally onto $\mathbf{x}$. Solve the following tasks:

- Formulate an algorithm based on Newton's method.
- How to initialize the algorithm?
- Implement your algorithm in Matlab. Define f (and its derivatives) using function handles, i.e., $f = @(u, v)...$ to write code that does not depend on the actual definition of f.
- Verify your solution graphically and computationally, i.e., plot the result and show that the difference vector is orthogonal to the tangent plane at the computed footpoint.

Problem 12. Consider the general **affine** registration problem: given two point sets $\{\mathbf{p}_i\}$ and $\{\mathbf{q}_i\}$ in correspondence, find the affine transformation

$$\mathbf{x} \mapsto \mathbf{a} + R\mathbf{x}$$

that minimizes $\sum w_i \|\mathbf{a} + R\mathbf{p}_i - \mathbf{q}_i\|^2$:

- Derive the corresponding linear system.
- Apply this to the data provided on TISS.
- Display the input point set $\{\mathbf{p}_i\}$ and $\{\mathbf{q}_i\}$ as well as the optimally transformed point set $\{\mathbf{a} + R\mathbf{p}_i\}$.

Problems due December 19, 2017.