Problem 16. Prove Euler's polyhedral formula

$$F - E + V = 2$$

for a spherical polyhedron with F faces, E edges, and V vertices. Hint: area formula for spherical polygons.

Problem 17. Write a Matlab program to compute the surface Φ enveloped by the 1-parameter family of planes

$$U(t, \mathbf{x}) = \mathbf{u}(t)^T \mathbf{x} - u_0(t), \qquad t \in [1/4, \pi - 1/4]$$

with

$$\mathbf{u}(t) = \begin{pmatrix} \cos t \cos \frac{(t-\pi)^2}{2\pi} \\ \cos t \sin \frac{(t-\pi)^2}{2\pi} \\ \sin t \end{pmatrix} \quad \text{and} \quad u_0(t) = 2\cos t + \frac{3}{2}.$$

Find the regression curve $\mathbf{c}$ and display Φ as a dense set of rulings along $\mathbf{c}$.

Problem 18. We consider the torus

$$\mathbf{x}(u, v) = \begin{pmatrix} (a + r\cos u)\cos v \\ (a + r\cos u)\sin v \\ r\sin u \end{pmatrix}, \qquad (u, v) \in [0, 2\pi] \times [0, 2\pi]$$

with positive a, r and $a > r$. Compute the tangent developable along surface curves $\mathbf{c}(u(t), v(t))$ with

1. $(u(t), v(t)) = (\pi/4, t)$
2. $(u(t), v(t)) = (t, 0)$
3. $(u(t), v(t)) = (t, t)$

It is sufficient to visualize rulings of the tangent developable, i.e., conjugate tangents along the curve $\mathbf{c}$.

Problems due January 25, 2018.