HECKIT SCHÄTZER Tobit V Modell

$$y_{i1}^* = x_{i1}\beta_1 + u_{i1}$$
$$y_{i2}^* = x_{i2}\beta_2 + u_{i2}$$
$$y_{i3}^* = x_{i3}\beta_3 + u_{i3}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \overset{iid}{\sim} N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \right)$$

$$d_i = \begin{cases} 1 & y_{i1}^* > 0 \\ 0 & y_{i1}^* \leq 0 \end{cases} \qquad y_i = \begin{cases} y_{i2}^* & d_i = 1 \\ y_{i3}^* & d_i = 0 \end{cases}$$

$$E(y_i \mid d_i = 1) = E(y_{i2}^* \mid y_{i1}^* > 0) = x_{i2}\beta_2 + \sigma_{12}\lambda(-x_{i1}\beta_1) \qquad \text{(Bsp 29)}$$

$$E(y_i \mid d_i = 0) = E(y_{i2}^* \mid y_{i1} \leq 0) = x_{i3}\beta_3 - \sigma_{13}\lambda(x_{i1}\beta_1) \qquad \text{(s.u.)}$$

$$Var(y_i \mid d_i = 1) = \sigma_2^2 - \sigma_{12}^2\,\delta(x_{i1}\beta_1) \qquad \text{(Bsp 29)}$$

$$Var(y_i \mid d_i = 0) = \sigma_3^2 - \sigma_{13}^2\,\delta(x_{i1}\beta_1) \qquad \text{(s.u.)}$$

SCHÄTZER :

a) Schätze β_1 durch $\hat{\beta}_{1,\text{PROBIT}}$

b) Schätze $\beta_2, \beta_3, \sigma_{12}, \sigma_{13}$ durch die OLS-Schätzer in:

(1) $y_i = x_{i2}\beta_2 + \sigma_{12}\lambda(-x_{i1}\hat{\beta}_{1,\text{PROBIT}}) + v_i \qquad i \in I_1 = \{i : d_i = 1\}$

(2) $y_i = x_{i3}\beta_3 - \sigma_{13}\lambda(x_{i1}\hat{\beta}_{1,\text{PROBIT}}) + w_i \qquad i \in I_0 = \{i : d_i = 0\}$

c) Schätze $\sigma_2^2 = Var(y_i \mid d_i = 1) + \sigma_{12}^2 \delta(x_{i1}\beta_1)$ und

$$\sigma_3^2 = Var(y_i \mid d_i = 0) + \sigma_{13}^2 \delta(x_{i1}\beta_1)$$

durch $\hat{\sigma}^2_{2,\,HECKIT} = \frac{1}{|I_1|} \sum_{i \in I_1} \left(\hat{v}_i^2 + \hat{\sigma}_{12}^2 \delta(x_i \hat{\beta}_{1,PROBIT}) \right)$ $\qquad \hat{v}_i$ ... Residuen aus (1)

und $\hat{\sigma}^2_{3,\,HECKIT} = \frac{1}{|I_0|} \sum_{i \in I_0} \left(\hat{w}_i^2 + \hat{\sigma}_{13}^2 \delta(x_i \hat{\beta}_{1,PROBIT}) \right)$ $\qquad \hat{w}_i$ ... Residuen aus (2)

Erweiterung zu Bsp. 29: $\binom{X}{Y} \sim N(\mu, \Sigma)$, $\quad \xi = Y - \mu_2 - \frac{\sigma_{12}}{\sigma_1^2}(X - \mu_1)$

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$\xi \sim N(0, \sigma_2^2 - \sigma_{12}^2/\sigma_1^2)$

$$E(Y \mid X \le 0) = E\left(\xi + \mu_2 + \frac{\sigma_{12}}{\sigma_1^2}(X - \mu_1) \,\Big|\, X \le 0\right)$$

$$= 0 + \mu_2 + \frac{\sigma_{12}}{\sigma_1^2}\left(\cancel{\mu_1} - \sigma_1 \lambda\left(\frac{\mu_1}{\sigma_1}\right)\right) - \frac{\sigma_{12}}{\sigma_1^2}\cancel{\mu_1}$$

$$= \mu_2 - \frac{\sigma_{12}}{\sigma_1} \lambda\left(\frac{\mu_1}{\sigma_1}\right)$$

$$Var(Y \mid X \le 0) = Var\left(\xi + \mu_2 + \frac{\sigma_{12}}{\sigma_1^2}(X - \mu_1) \,\Big|\, X \le 0\right)$$

$$= Var\left(\xi + \frac{\sigma_{12}}{\sigma_1^2} \,\Big|\, X \le 0\right)$$

$$= \sigma_2^2 - \sigma_{12}^2/\sigma_1^2 + \frac{\sigma_{12}^2}{\sigma_1^{\cancel{4}\,2}} \cancel{\sigma_1^2} \left(1 - \delta\left(\frac{\mu_1}{\sigma_1}\right)\right)$$

$$= \sigma_2^2 - \frac{\sigma_{12}^2}{\sigma_1^2} \delta\left(\frac{\mu_1}{\sigma_1}\right)$$