alternatives or sequencing of decisions. In practice many different multinomial models are used.

Section 15.2 presents an application to illustrate the issues discussed in this chapter. General results for multinomial models are given in Section 15.3. The conditional and multinomial logit models are presented in Section 15.4. The additive random utility model is presented in Section 15.5. The nested logit, random parameters logit, and multinomial probit models are the subject of Sections 15.6–15.8. Ordered and sequential models are detailed in Section 15.9. Multivariate models with more than one discrete outcome variable are presented in Section 15.10. Semiparametric estimators are briefly reviewed in Section 15.11.

15.2. Example: Choice of Fishing Mode

This section illustrates multinomial logit, the simplest unordered multinomial model, and variations detailed in Section 15.4 that permit regressors to vary across alternatives. The emphasis is on interpretation of estimated models. The marginal effect of a change in a regressor is more complicated than the usual impact on a single conditional mean. For multinomial data there is instead a separate marginal effect on the probability of each outcome, and these marginal effects sum to zero since probabilities sum to one.

The application is to choice of fishing mode. The dependent variable y takes value 1, 2, 3, or 4 depending on which of the four mutually exclusive alternative modes of fishing – respectively, beach, pier, private boat, and charter boat – is chosen. An unordered multinomial model such as multinomial logit is appropriate, since there is no clear ordering of the outcome variable. Regressors are individual income, which does not vary with fishing mode, and price and catch rate, which do vary by fishing mode and across individuals.

The sample of 1,182 people comes from a survey conducted by Thomson and Crooke (1991) and analyzed by Herriges and Kling (1999). The data are summarized in Table 15.1, which gives averages for the subsamples of people who chose each of the modes as well as the overall sample average of regressors.

15.2.1. Conditional Logit: Alternative-Varying Regressors

First consider the role of price and catch rate, regressors that vary across alternatives except that for these data the price of beach and pier fishing are the same.

Looking down the columns of Table 15.1, we see that people tend to fish where it is cheapest for them to do so. For example, for people choosing to fish from the beach the average price was $36 compared to average prices of $36, $98, and $125 for the other modes. More generally, for people choosing the beach and pier these modes were on average much cheaper than the boat modes, and for people fishing from a boat this was on average much cheaper than beach or pier fishing. The relationship between mode choice and catch rate is less clear-cut, though it is clear that the charter boat has the highest catch rate.

Table 15.1. *Fishing Mode Multinomial Choice: Data Summary*

	Sub sample Averages				
	$y = 1$	$y = 2$	$y = 3$	$y = 4$	**All y**
Explanatory Variable	**Beach**	**Pier**	**Private**	**Charter**	**Overall**
Income ($1,000s per month)	4.052	3.387	4.654	3.881	4.099
Price beach ($)	36	31	138	121	103
Price pier ($)	36	31	138	121	103
Price private ($)	98	82	42	45	55
Price charter ($)	125	110	71	75	84
Catch rate beach	0.28	0.26	0.21	0.25	0.24
Catch rate pier	0.22	0.20	0.13	0.16	0.16
Catch rate private	0.16	0.15	0.18	0.18	0.17
Catch rate charter	0.52	0.50	0.65	0.69	0.63
Sample probability	0.113	0.151	0.354	0.382	1.000
Observations	134	178	418	452	1182

For alternative-specific regressors that vary across alternatives, such as price and catch rate, the multinomial logit model is called a conditional logit model (see Section 15.4.1). The probability of the ith individual choosing fishing mode j is given by

$$p_{ij} = \Pr[y_i = j] = \frac{\exp(\beta_P P_{ij} + \beta_C C_{ij})}{\sum_{k=1}^{4} \exp(\beta_P P_{ik} + \beta_C C_{ik})}, \quad j = 1, \ldots, 4,$$

where P denotes price, C denotes catch rate, the subscript i denotes the ith individual, and subscript j or k denotes the alternative. This model is an obvious extension of binary logit and gives probabilities that lie between 0 and 1 and sum to one. Other multinomial models use a different functional form for p_{ij}.

The coefficient estimates are given in the CL column of Table 15.2. For the CL model, though not for all multinomial models, the sign of the coefficient is directly interpretable. Anticipating results from Section 15.4.3, since $\beta_P < 0$ we have that an increase in the price of one alternative decreases the probability of choosing that alternative and increases the probability of choosing other alternatives. Similarly, since $\beta_C > 0$, an increase in the catch rate for one alternative increases choice probability for that alternative and decreases the choice probability for other alternatives.

A standard measure of the impact of changes in regressors is $N^{-1} \sum_{i=1}^{N} \partial p_{ij}/\partial x_{ikr}$, the average marginal response of the probability of choosing alternative j when the rth regressor increases by one unit for alternative k and is unchanged for the other alternatives. For the CL model this is estimated by $N^{-1} \sum_{i=1}^{n} \widehat{p}_{ij}(\delta_{ijk} - \widehat{p}_{ik})\widehat{\beta}_r$ (see (15.18)), where $\widehat{\boldsymbol{\beta}}$ is the estimate of $\boldsymbol{\beta}$ and $\widehat{p}_{ij}$, $j = 1, \ldots, m$, are the predicted probabilities.

The average responses across the four modes for the two regressors price and catch rate are given in Table 15.3. The table gives the effect on choice probability of a 100-unit (or $100) change in price and the effect of a one-unit change in the catch rate. For example, an increase of $100 in the price of beach fishing leads to a decrease of 0.272

Table 15.2. *Fishing Mode Multinomial Choice: Logit Estimates*[a]

			Model type		
Regressor	**Type**	**Coefficient**	**CL**	**MNL**	**Mixed**
Price (P)	Specific	β_P	−0.021	–	−0.025
Catch rate (C)	Specific	β_{CR}	0.953	–	0.358
Intercept	Invariant	α_1 : *Beach*	–	0.0	0.0
		α_2 : *Pier*	–	0.814	0.778
		α_3 : *Private*	–	0.739	0.527
		α_4 : *Charter*	–	1.341	1.694
Income (I)	Invariant	β_{I1} : *Beach*	–	0.0	0.0
		β_{I2} : *Pier*	–	−0.143	−0.128
		β_{I3} : *Private*	–	0.092	0.089
		β_{I4} : *Charter*	–	−0.032	−0.033
− ln L			−1311	−1477	−1215
Pseudo-R^2			0.162	0.099	0.258

[a] Type of regresssor is alternative-specific (price and catch rate) or alternative-invariant (income). Outcomes are (1) beach, (2) pier, (3) private, and (4) charter. MLE estimates are for conditional logit (CL), multinomial logit (MNL), and mixed logit (Mixed) models. MNL and Mixed models are normalized to base category beach. All estimates except that for β_{I4} are statistically significant at 5%.

in the probability of fishing and an increase of 0.119, 0.080, and 0.068, respectively, in the probability of fishing from a beach, a pier, a private boat, and a charter boat. Note that the changes in probabilities sum to zero, as expected.

Calculation of these marginal effects and probabilities requires postestimation computation. A back-of-the-envelope calculation uses $\bar{p}_j(\delta_{jk} - \bar{p}_k)\widehat{\beta_r}$ for the CL model, where $\bar{p}_j$ is the sample average probability. For the effect of a \$100 change in the price of beach fishing on the probability of beach fishing this yields $100 \times 0.113(1 - 0.113) \times (-0.021) = -0.21$, compared to the sample average value of −0.272 in the table. This approximation becomes less reasonable as probabilities get closer to 0 or 1.

The results in Table 15.3 are consistent with the view that the greatest substitution is between pier and beach fishing and between private boat and charter boat

Table 15.3. *Fishing Mode Choice: Marginal Effects for Conditional Logit Model*[a]

	\$100 Change in Price of				One-Unit Change in Catch Rate for			
	Beach	**Pier**	**Private**	**Charter**	**Beach**	**Pier**	**Private**	**Charter**
Change in Pr[beach]	−.272	.119	.085	.068	.126	−.055	−.040	−.032
Change in Pr[pier]	.119	−.263	.080	.064	−.055	.122	−.037	−.030
Change in Pr[private]	.080	.080	−.391	.225	−.040	−.037	.182	−.105
Change in Pr[charter]	.068	.064	.225	−.357	−.032	−.030	−.105	.166

[a] Average marginal response of the probability of choosing each alternative when a regressor changes for one of the alternatives and is unchanged for the other alternatives.

fishing. Specifically, price increases, or catch rate decreases, for pier lead to substitution to beach, and vice versa. A similar result holds for charter versus private boat.

These choice probability changes are for large changes in the regressors, given that average price is \$86 and average catch rate is 0.30. One can instead calculate elasticities. Elasticities for choice probabilities need to be used with care, however, because probabilities are bounded between 0 and 1. A change in predicted probability from 0.01 to 0.02 will lead to an elasticity roughly 50 times larger than that for a change in predicted probability from 0.50 to 0.51.

15.2.2. Multinomial Logit: Alternative-Invariant Regressors

Now consider the role of income, measured as monthly income in thousands of dollars. From Table 15.1 it appears that as income rises the fishing mode moves progressively from pier, where average monthly income of people fishing at a pier is \$3,387, to charter boat to beach and finally to private boat, where the average income is \$4,654.

Because income is invariant across alternatives the appropriate model is the multinomial logit model (presented in Section 15.4.1). This lets regressor coefficients vary across alternatives, with

$$p_{ij} = \Pr[y_i = j] = \frac{\exp(\alpha_j + \beta_{Ij} I_i)}{\sum_{k=1}^{4} \exp(\alpha_k + \beta_{Ik} I_i)}, \quad j = 1, \ldots, 4,$$

where I denotes income. A normalization of parameters is needed as a consequence of the restriction that probabilities sum to one. The empirical results set $\alpha_1 = 0$ and $\beta_{I1} = 0$.

The parameter estimates are given in the MNL column of Table 15.2. Coefficient interpretation is more difficult than for the CL logit model. In particular, for MNL models a positive regression parameter does not mean that an increase in the regressor leads to an increase in the probability of that alternative. Instead, interpretation for the MNL model is relative to the reference or base category group, here beach as the beach coefficients were normalized to zero. Compared to beach fishing a higher income leads to reduced likelihood of fishing from a pier (since $\beta_{I2} = -0.143 < 0$) or a charter boat (since $\beta_{I4} = -0.032$) and greater likelihood of use of a private boat (since $\beta_{I3} = 0.092$).

The magnitude of the response to income changes can be measured using $N^{-1} \sum_{i=1}^{N} \partial p_{ij}/\partial I_i$, the marginal effect averaged over individuals. For the MNL models this is estimated by $N^{-1} \sum_{i=1}^{N} \widehat{p}_{ij}(\widehat{\beta}_j - \widehat{\bar{\beta}}_i)$ (see (15.19)), where $\widehat{\boldsymbol{\beta}}_j$ is the estimate of $\boldsymbol{\beta}_j$, $\bar{\boldsymbol{\beta}}_i = \sum_{l=1}^{m} p_{il} \boldsymbol{\beta}_l$ is a probability weighted average of the $\boldsymbol{\beta}_l$, and $\widehat{p}_{ij}$, $j = 1, \ldots, m$, are the predicted probabilities. For the four choices a \$1,000 increase in monthly income is associated with changes of 0.000, −0.021, 0.033, and −0.012 in, respectively, the probabilities of fishing from beach, pier, private boat, and charter boat. This indicates little change in beach fishing, movement out of pier and charter boat fishing, and movement to private boat fishing. Since average monthly income is \$4,100 the changes in probability are of reasonable size.