

# Q.O.M. – Model-based Decision Support

Exam 9 (home assignment)

till June 23, 2016

## Meat Vendor Problem

There are five beef supply vendors (v) and two distribution centers (d). We want to minimize costs associated with the production of three beef products (p) and delivery of these beef products to distribution centers while satisfying the demands of the distribution centers. Figure 1 shows a conceptual diagram (network) of this transportation and distribution problem. Where

$costD(v,d,p)$	Cost of shipment from vendor to distribution center
$xD(v,d,p)$	Product shipped from vendor to distribution center
$yD(v,d,p)$	Binary variable for product shipped
$ProdP(v,p)$	Beef production of p at vendor v
$costV(v)$	Cost driven by beef production
$yP(v,p)$	Binary variable of beef product
$yV(v)$	Binary variable of vendor
$dcdemand(p,d)$	Demand of product at distribution center

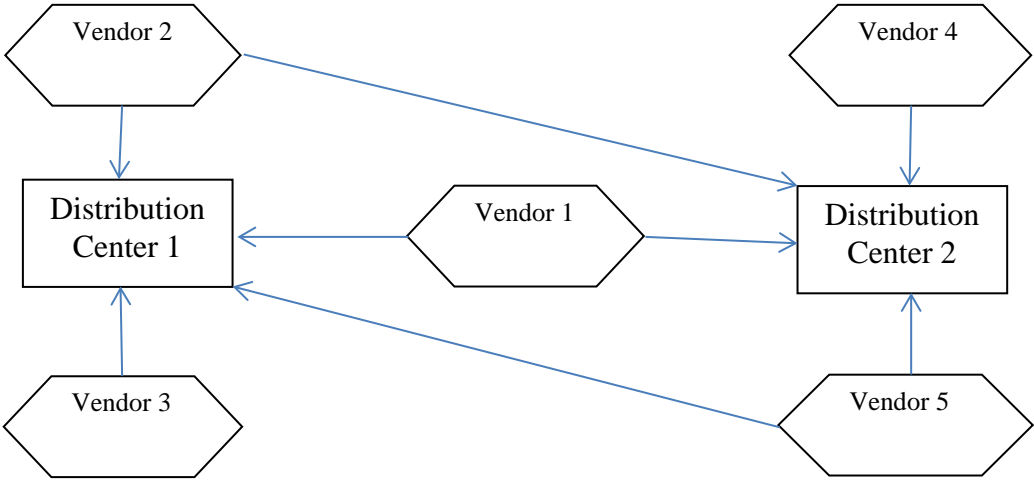


Figure 1: Possible supplier – customer allocation (supply chain distribution)

Table 1 shows the expected demand per year:

product / distr	Distr. Center 1	Distr. Center 2
<b>fillet</b>	1,720,000	810,000
<b>minced (beef)</b>	11,190,000	480,000
<b>boiled (beef)</b>	3,570,000	0

Table 1:  $dcdemand(p,d)$ ; demand in lb per year

However, there is a 25% uncertainty in demand. We model deviance from expected demand by a normal distributed random variable with mean 0 and standard deviation  $\sigma = 0.08897$ . We motivate  $\sigma$  by the 0.9975 quantile of  $N(0,\sigma)$  at 0.25 (that is the deviance is between -25% and 25% with a probability of 99,75%).

Further, find in Table 2 the annual costs ensuring the continuity of services at vendor  $v$  (if no production is scheduled there are no costs). In other words, if we decide that vendor  $v$  processes meat, then we will have to include costs  $costV$  (in million \$) to the cost minimization problem (these “fix” costs are independent from the beef product and the amount processed).

v	1	2	3	4	5
costV	0.8067	0.8427	0.8151	0.8073	0.8048

Table 2: costV; annual costs in Million US\$ for operating vendor  $v$ .

Production and transport costs per produced unit meat (costD) are given in Table 3.

p = 1 (fillet)			p = 2 (minced)			p = 3 (boiled)		
v \ d	Distr 1	Distr 2	v \ d	Distr 1	Distr 2	v \ d	Distr 1	Distr 2
1	43.1	6.5	1	25.5	75.9	1	12.7	21.2
2	36.3	87.1	2	64.7	18.0	2	84.0	75.9
3	43.4	11.7	3	29.5	37.3	3	60.7	64.8
4	22.2	15.3	4	58.5	6.5	4	19.8	49.2
5	9.5	79.7	5	12.1	34.2	5	44.0	38.2

Table 3: costD; production plus transportation costs per lb meat in US\$Cent.

One family of decisions is, which of the five vendors should operate, hence process meat. These decisions are strongly related to the family of decisions which vendor supplies which distribution center, because only those vendors, who supply a distribution center, will operate.

For each individual beef product and each individual distribution center, exactly one vendor can be chosen as the supplier (for different distribution centers or different beef products different vendors may be chosen). With this restriction, the amount processed at individual vendors will be determined, if vendor – distribution center allocation is decided (the allocated vendor has to process and transship exactly the demand).

### Wait and See solution

In this case study, demand is uncertain and perfect information is a realization of the demand. Assuming to know the realization of the demand, we can use mathematical programming computing the optimal vendor – distribution center allocation. Of course, there is not a single general solution, but for each realization of the demand we get a specific solution.

If we were in the favorable position to postpone decision making after the realization of demand, we would be able to do decision making under perfect information. It does not mean that we enumerate all possible realization of demand and list the corresponding optimal allocation. However the optimal decisions (allocations) (and as a consequence the resulting

costs) are random variables themselves governed by the distribution of the random variable demand. What we are interested in, if we were able to postpone decision making, would be the expected (optimal) costs of a implementation of such a “Wait and See” solution; we call this value the expected payoff of perfect information. This concept was published by TU Wien Professor Tintner 1955. The EPPI is the best what you can achieve, and EPPI concept is still a very important concept; at least for benchmarking.

In the home assignment, we take a normal distributed demand as a basis. We are not able to compute EPPI based on the normal distribution function, but instead you should compute a sample mean as a proxy for EPPI. For computing the sample, GAMS provides a suitable normal distributed random number generator for realization of demand and, of course, GAMS provides the optimization solver(s).

Try to understand the model „Supply Chain“ in the GAMS Code „MeatVendor.gms“ that you will find at TISS. The computation of EPPI (more correct: mean sample as proxy for the EPPI) is already available there for uncertainty of a deviance between -25% and 25%. Additionally compute the mean sample proxy of EPPI in case of (-20%,20%) and (-30%, 30%) deviance (you have to work with 0.9975 quantile of a normal distribution  $N(0,\sigma)$ , hence have to update  $\sigma$ ).

## Here and Now solution

Rather it is likely that we have to make your decision before any realization of the demand. Conceptual, we want to make decisions (vendor – center allocation), which “minimizes on average the costs for all possible demand realizations”. For a given (fixed) vendor – center allocation, we compute the expected costs (based on the distribution of the uncertain demand). Next, we choose in the set of all feasible vendor-center allocations that allocation with the lowest expected costs as the solution for the problem.

The problem of this concept is that we need a mapping of the space of feasible decisions into the set of real numbers (costs) based on the expected costs. Theoretically, we can enumerate all possible decisions (in our case 125 feasible combinations of product – vendor – center; in the setup of Figure 1 we have 64 feasible combinations), compute individual expected costs and compare. Alternatively, we can compute some individual feasible combinations expected costs and extrapolate the remaining by function fitting or interpolation. (Remark: another method is to investigate the space of feasible decisions governed by Simulated Annealing or other heuristic methods – this method is often referred as Stochastic Optimization; but take care “Stochastic Optimization” is sometimes also used as a synonym for Stochastic Programming.)

As mentioned, you should not enumerate all possible combinations. For the home assignment choose 3-5 combinations, compute in GAMS („MeatVendor.gms“) expected costs and document the combinations and their individual expected costs for submission.

You can do the home assignment in groups; in this case each member of the group should choose 3-5 different combinations (this means some coordination). In the submission, you should document who has investigated which combinations in GAMS („MeatVendor.gms“). We'll see who will have found the best.

In case that there are any obscurities do not hesitate to contact me.