Game-theoretic Modeling **ÜBUNG** **WS 2018**

11. (Costs allocation problem:) Three airlines plan to build a new airstrip. The airlines operate planes with different sizes. The first airline operates smaller planes and a shorter airstrip with construction costs of $c_1 = 10M$ would be sufficient. The second airline needs at least a medium length airstrip, which costs $c_i = 18M$. The third airline operates huge jumbo jets and has a need of a really long airstrip, which costs $c_i = 33M$.

    If airlines form a coalition, they will build a long enough airstrip so that all their planes can land. If there isn't any agreement, each airline builds its own airstrip.

    Formulate a coalition game and compute the Shapley values in order to decide how they should finance the construction in case of a grand coalition (Hint: cost function $c(S) = \max_{i \in S} c_i$. Excel sheet discussed in the lecture is available at TISS.)

12. A player who is in every winning coalition in a simple game is called a veto player.

    - N = $\{1, 2, 3\}$ and let $v(\{1,2,3\}) = 1$ and $v(S) = 0$ for all other coalitions S. This is a simple game and in this game every player is a veto player. How does the core of this coalition game looks like?
    - N = $\{1, 2, 3\}$ and let $v(\{1,2,3\}) = v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1$, and $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. This is a simple game and in this game there are no veto players. How does the core of this coalition game looks like?
    - N = $\{1, 2, 3\}$ and let $v(\{1,2,3\}) = v(\{1,2\}) = v(\{1,3\}) = 1$, and $v(\{2,3\}) = v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. This is a simple game and Player 1 is the only veto player. How does the core of this coalition game looks like?

13. Prove: A player who is in every winning coalition in a simple game is called a veto player.A simple game has a non-empty core if and only it has veto players.

14. Hillary and Barack share 1000USD according to the following rules. First Hillary suggests an amount $x \geq 0$; this is the amount she wants. Barack, having heard Hilary's suggestion, may propose an amount $y \geq 0$. If the sum of x and y is greater than 1000USD, then the payoff of the game is equal to $(0, 0)$. Otherwise, Hillary receives the desired amount x and Barack the desired amount y.

    - Draw a suitable game tree for this set-up.
    - Alternatively, formulate this division problem as a bargaining game (F, d).

Now Angela plays with Donald the same game as Hillary and Barack.However, Angela has to tax her profits with 50% income tax. Donald does not have to pay any income tax due to losses in other businesses. Formulate this division problem as a bargaining game (F, d).

15. Two players have to agree on the devision of one unit of a perfectly divisible good, say a liter of wine. If they reach an agreement, say (α, β) where $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$, then they split up the one unit accordingly to this agreement; otherwise, they both receive nothing (disagreement point (0,0)). The players have preferences for the good, described by utility functions $u_i(\cdot)$.

    Assume that Player I's utility function is $u_1(\alpha) = \alpha$ and Player II's utility function is $u_2(\beta) = 2\beta - \beta^2$. Formulate the optimization problem in division of the good (variable α) and – alternatively – formulate the Nash bargaining problem (optimization in utilities).