25. In this game in a strategic setting, players 1 and 2 play according to the matrix shown. However, player 1's payoff number x is private information. Player 2 knows only that x = 12 with probability $\frac{2}{3}$ and $x = 0$ with probability $\frac{1}{3}$.

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} x, 9 & 3, 6 \\ 6, 0 & 6, 9 \end{pmatrix} \end{array}$$

Note that the matrix pictured is not the true normal form of the game, because player 1 observes $x$ before making his decision. Player 1 observes nature's action before selecting between $U$ and $D$, yet player 2 must make her choice without observing player 1's type or action.

- Draw the game tree
- Determine the information sets
- Compute for all 4 pure strategies of player 1 beliefs for player 2
- and picture the normal form of this game
- Determine the pure Nash equilibrium
- Is this Nash equilibrium perfect Bayesian?

26. The beer-quiche game. Consider the following signaling game. Player 1 is either "weak" or "strong". This is determined by a choice move, resulting in player 1 being "weak" with with probability $\frac{1}{10}$. Player 1 is informed about the outcome of this chance move but player 2 is not; but the probabilities of either type of player 1 are common knowledge among the two players. Player 1 has two actions: either have quiche (Q) or have beer (B) for breakfast. Player 2 observes the breakfast of player 1 and then decides to duel (D) or not to duel (N) with player 1. The payoffs are as follows. If player 1 is weak and eats quiche then D and N give him payoffs of 1 and 3, respectively, if he is weak and drinks beer, the these payoffs are 0 and 2, respectively. If player 1 is strong, then the payoffs are 0 and 2 from D and N, respectively, if he eats quiche; and 1 and 3 from D and N, respectively, if he drinks beer. Player 2 has payoff 0 from not dueling, payoff 1 from dueling with the weak player 1, and payoff -1 from dueling with the strong player 1.

- Draw the game tree
- Compute all pure strategy Nash equilibria of the game
- Find out which of these Nash equilibria are perfect Bayesian equilbria

- Give the corresponding beliefs and determine whether these equilibria are pooling or separating.

27. Compute the evolutionary stable strategies for the following payoff matrix

$$A = \begin{array}{c} \\ F \\ B \end{array} \begin{array}{c} \begin{array}{cc} F & B \end{array} \\ \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

28. Determine the replicator dynamics for the Rock-Paper-Scissors Game

$$\begin{array}{c} \\ Rock \\ Paper \\ Scissors \end{array} \begin{array}{c} \begin{array}{ccc} Rock & Paper & Scissors \end{array} \\ \begin{pmatrix} 1 & 0 & a \\ a & 1 & 0 \\ 0 & a & 1 \end{pmatrix} \end{array}$$

29. For every real number $a$ the 3-person TU-game $v_a$ is given by $v_a\{i\} = 0, i = 1, 2, 3, v_a\{1, 2\} = 3, v_a\{1, 3\} = 2, v_a\{2, 3\} = 1$, and $v_a\{1, 2, 3\} = a$.

    - Determine the minimal value of $a$ so that the TU-game $v_a$ has a nonempty core.
    - Calculate the Shapley value of $v_a$ for $a = 6$.
    - Determine the minimal value of $a$ so that the Shapley value of $v_a$ is a core distribution (allocation).

Please note and remember your working time for the preparation of example 25 to 29.