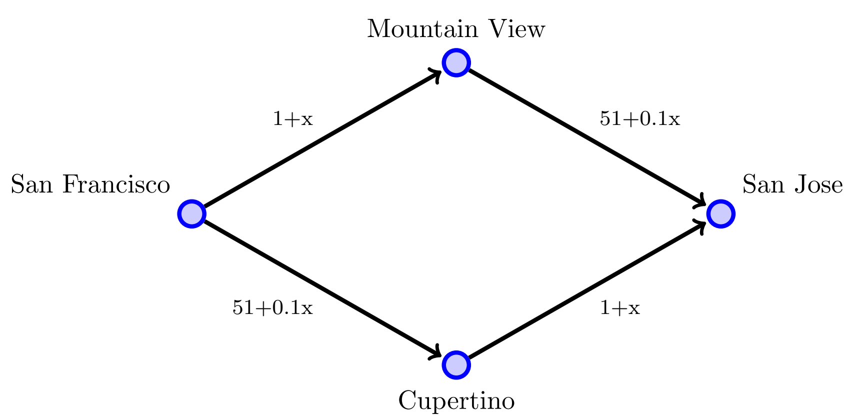
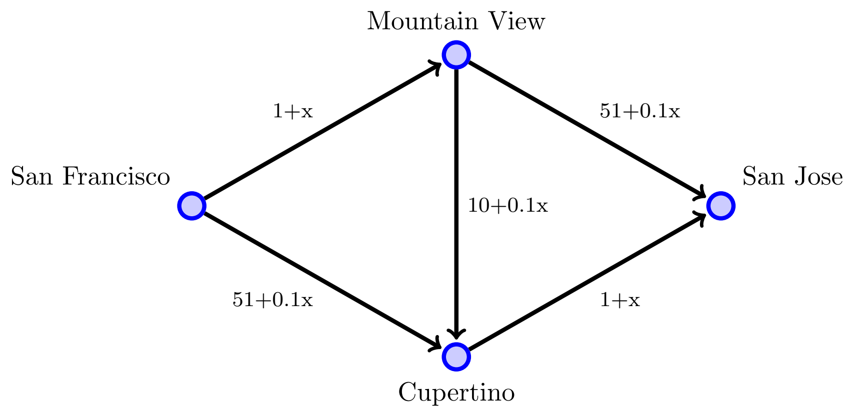
21. Braess Paradoxon. Consider a finite game with 60 players. There are two main roads connecting San Francisco and San Jose, a northern road via Mountain View and a southern road via Cupertino. Travel time on each of the roads depend on the number of x of cars using the road per minute, as indicated in the following diagram:

    For example, the travel time between San Francisco and Mountain View is 1 + x, where x is the number of cars per minute using the road connecting these cities, and the travel time between Mountain View and San Jose is 51 + 0.1x, where x is the number of cars per minute using the road connecting those two cities. Each driver chooses which road to take in going from San Francisco to San Jose, with the goal of reducing to a minimum the amount of travel time. Early in the morning, 60 cars per minute get on the road from San Francisco to San Jose (where we assume the travelers leave early in the morning so that they are the only ones on the road).



    What are all the Nash equilibria of this game? At these equilibria, how much time does the trip take at an early morning hour?

    The California Department of Transportation constructs a new road between Mountain View and Cupertino, with travel time between these cities 10 + 0.1 x. This road is one way, enabling travel solely from Mountain View to Cupertino.



Find a Nash equilibrium in this new game. Under this equilibrium how much time does it make to get to San Jose from San Francisco on an early morning hour? Does the construction of the additional road improve travel time?

Such a phenomenon was in fact noted in New York (where the closure of a road for construction work had the effect of decreasing travel time) and in Stuttgart (where the opening of a new road increased travel time).

22. Bidding Game. Players 1 and 2 bid for an object that has value 2 for each of them. They both have wealth 3 and are not allowed to bid higher than this amount. Each bid must be a nonnegative integer amount. Besides bidding, each player, when it is his turn, has the option to pass (P) or to match (M) the last bid, where the last bid is set at zero at the beginning of the game. If a player passes (P), then the game is over and the other player gets the object and pays the last bid. If a player matches (M), then the game is over and each player gets the object and plays the last bid with probability $\frac{1}{2}$. Player 1 starts, and the players alternate until the game is over. Each new bid must be higher than the last bid.

    - Draw the game tree of this extensive form game
    - How many strategies does player 1 have? Player 2?
    - How many subgame perfect equilibria does this game have? What is (are) the possible subgame perfect equilibrium outcome(s)?

23. Repeated Games. Consider the Prisoners' Dilemma Game. Suppose this game is played twice. After the first play of the game the players learn the outcome of that play. The final payoff for each player is the sum of the payoffs of the two stages.

    - Write down the extensive form of this game. How many strategies does each player have? How many subgames does this game have?

- Determine the subgame perfect equilibrium or equilibria of this game. Do have an idea, how subgame perfect equilibrium would look like, if the game is repeated more than twice.

24. Ultimatum offer bargaining game. Two players negotiate over the price of a painting that player 1 sells to player 2. Player 1 proposes a price p to player 2. Then, after observing player 1's offer, player 2 decides whether to accept it (yes) or reject it (no). If player 2 accepts the proposal, then player 1 obtains p and player 2 obtains 100-p. If player 2 rejects the proposal, then each player gets zero.

    In this game, there are uncountable many trivial information sets for player 2. How does the set of actions look like in such an information set? Player 1 has uncountable many actions in her unique information set. However, the principle of Nash / subgame perfectness - discussed in finite games - can in a similar way applied to this infinite bargaining game.

    A cutoff-rule strategy for player 2 is a strategy where player 2 accepts any offer up to a $\underline{p}$, and rejects any offer higher than this threshold $\underline{p}$. Discuss, if a given offer $\hat{p} < 100$, let say $\hat{p} = 50$, combined with the cutoff-rule strategy with threshold $\underline{p} = \hat{p}$ defines a Nash equilibrium.

    Verify that there is a subgame perfect Nash equilibrium in which player 1 offers $p^* = 100$ and player 2 has the strategy of accepting any offer $p \leq 100$ and rejecting any offer $p > 100$.

Please note and remember your working time for the preparation of example 21 to 24.