1. (Inspector Game): During the 1960s, within the framework of negotiations between the United States (US) and the Union of Soviet Socialist Republics (USSR) over nuclear arms limitations, a suggestion was raised that both countries commit to a moratorium on nuclear testing. One of the objections to this suggestion was the difficulty in supervising compliance with such a commitment. Detecting above-ground nuclear tests posed no problem, because it was easy to detect the radioactive fallout from a nuclear explosion conducted in the open. This was not true, however, with respect to underground tests, because it was difficult at the time to distinguish seismographically between an underground nuclear explosion and an earthquake. The US therefore suggested that in every case of suspicion that a nuclear test had been conducted, an inspection team be sent to perform on-site inspection. The USSR initially objected, regarding any inspection team sent by US as a potential spy operation. At later stages in the negotiations, Soviet negotiators expressed readiness to accept three on-site inspections annually, while American negotiators demanded at least eight on-site inspections. The expected number of seismic events per year considered sufficiently strong to arouse suspicion was 300.

   The model presented in this exercise assumes the following:

   - The USSR can potentially conduct underground nuclear tests on one of two possible distinct dates, labeled A and B, where B is the later date.
   - The USSR gains nothing from choosing one of these dates over the other for conducting an underground nuclear test, and the US loses nothing if one date is chosen over another.
   - The USSR gains nothing from conducting nuclear tests on both of these dates over its utility from conducting a test on only one date, and the US loses nothing if tests are conducted on both dates over its utility from conducting a test on only one date.
   - The US may send an inspection team on only one of the two dates, A or B, but not on both.
   - The utilities of the two countries from the possible outcomes are:
     - If the Partial Test Ban Treaty (PTBT) is violated by the USSR and the US does not send an inspection team, the US receives -1 and the USSR receives 1.
     - If the PTBT is violated by the USSR and the US sends an inspection team, the US receives $-\alpha$ and the USSR receives $-\beta; \alpha > 0$ and $0 < \beta < 1$.
     - If the PTBT is not violated both US and USSR receive 0.

As preparations for tests and inspection have to be done in advance, assume a simultaneous game. Formulate a bi-matrix game. Argue why USA never gave up testing.

2. Solve the following bi-matrix game by iterative removal of dominated actions:

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} 3,3 & 0,5 & 0,4 \\ 0,0 & 3,1 & 1,2 \end{pmatrix} \end{array}$$

Is the solution a saddle point?

3. Strategies that "survive" iterative removal of dominated strategies are called rationalizable strategies. Find the set of rationalizable strategies (Hint: start with a mixed strategy of Player II's actions Y and Z):

$$\begin{array}{c} \\ U \\ M \\ L \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} 5,1 & 0,4 & 1,0 \\ 3,1 & 0,0 & 3,5 \\ 3,3 & 4,4 & 2,5 \end{pmatrix} \end{array}$$

4. The normal-form game pictured below represents a situation in tennis, whereby the server (Player I) decides whether to serve to the opponent's forehand (F), center (C), or backhand (B) side. Simultaneously, the receiver (Player 2) decides whether to favor the forehand, center, or backhand side.

$$\begin{array}{c} \\ F \\ C \\ B \end{array} \begin{array}{c} \begin{array}{ccc} F & C & B \end{array} \\ \begin{pmatrix} 0,5 & 2,3 & 2,3 \\ 2,3 & 0,5 & 3,2 \\ 5,0 & 3,2 & 2,3 \end{pmatrix} \end{array}$$

Mixed strategies in Player I's action (C) and (B) dominates action (F). Compute all in actions (C) and (B) mixed strategies, which dominate (F). (Hint: compute the interval of p – the probability that Player I plays the (C) action.)

5. Suppose that two people decide to form a partnership firm. The revenue of the firm depends on the amount of effort expended on the job by each person and is given by:

$$r(e_1, e_2) = a_1 e_1 + a_2 e_2,$$

where e_1 is the effort level of person 1 and e_2 is the effort level of person 2. The numbers a_1 and a_2 are positive constants. The contract that was signed by the partners stipulates that person 1 receives a fraction t (between 0 and 1) of the firm's revenue, and person 2 receives a $1 - t$ fraction. That is, person 1 receives the amount $t\,r(e_1, e_2)$, and person 2 receives $(1-t)\,r(e_1, e_2)$. Each person dislikes effort, which is measured by a personal cost of e_1^2 for person 1 and e_2^2 by person 2. Person i's utility in this endeavor is the amount of revenue that this person receives, minus the effort cost e_i^2. The effort levels (assumed to be nonnegative) are chosen by the people simultaneously and independently.

- Note down the strategic (normal) form of this game.
- Using dominance, compute the strategies that the players rationally select (as functions in t, a_1, and a_2).

6. Consider a version of the Cournot duopoly game, where firms 1 and 2 simultaneously and independently select quantities to produce in a market. The quantity selected by firm i is denoted q_i and must be greater than or equal zero, for i = 1,2. The market price is given by $p = 100 - 2q_1 - 2q_2$. Suppose that each firm produces at a cost of 20 per unit. Further, assume that each firm's payoff is defined as its profit. Suppose that the player 1 has the belief that player 2 is equally likely to select each of the quantities 6, 11, and 13. What is player 1's expected payoff of choosing a quantity of 14?

7. Find for the following bi-matrix game all Nash equilibria:

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{pmatrix} 0,0 & 2,1 \\ 3,2 & 1,2 \end{pmatrix} \end{array}$$

   Next solve this game by iterated removal of dominated strategies. What happens to the Nash equilibria?

8. Consider a sequential Cournot duopoly game, based on the inverse price function $p = 2 - 3Q$ (or zero, whichever is larger), where Q is the total quantity produced. Assume that Player I is the follower and compute the Stackelberg equilibrium.

9. Suppose a manager and a worker interact as follows. The manager decides whether to hire or not hire the worker. If the manager does not hire the worker, then the game ends. When hired, the worker chooses to exert either high effort or low effort. On observing the worker's effort, the manager chooses to retain or fire the worker.

   - Is "not hire" an action or a strategy?
   - Note down all strategy profiles of this game.

10. Represent the following game in extensive form (game tree). Firm A decides whether to enter firm B's industry. Firm B observes this decision. If firm A enters, then the two firms simultaneously decide whether to advertise. Otherwise, firm B alone decides whether to advertise. With two firms in the market, the firms earn profits of \$3 million each if they both advertise and \$5 million if they both do not advertise. If only one firm advertises, then it earns \$6 million and the other earn \$1 million. When firm B is solely in the industry, it earns \$4 million if it advertises and \$3.5 million if it does not advertise. Firm A earns nothing if it does not enter.