11. (Costs allocation problem:) Three airlines plan to build a new airstrip. The airlines operate planes with different sizes. The first airline operates smaller planes and a shorter airstrip with construction costs of $c_1 = 10M$ would be sufficient. The second airline needs at least a medium length airstrip, which costs $c_i = 18M$. The third airline operates huge jumbo jets and has a need of a really long airstrip, which costs $c_i = 33M$.

    If airlines form a coalition, they will build a long enough airstrip so that all their planes can land. If there isn't any agreement, each airline builds its own airstrip.

    Formulate a coalition game in order to decide how they should finance the construction in case of a grand coalition (Hint: cost function $c(S) = \max_{i \in S} c_i$.)

    A MS Excel sheet is available at TISS - try to understand the formal computation of the Shapley values.

12. A player who is in every winning coalition in a simple game is called a veto player.

    - N = {1, 2, 3} and let $v(\{1, 2, 3\}) = 1$ and $v(S) = 0$ for all other coalitions S. This is a simple game and in this game every player is a veto player. How does the core of this coalition game looks like?
    - N = {1, 2, 3} and let $v(\{1, 2, 3\}) = v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1$, and $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. This is a simple game and in this game there are no veto players. How does the core of this coalition game looks like?
    - N = {1, 2, 3} and let $v(\{1, 2, 3\}) = v(\{1, 2\}) = v(\{1, 3\}) = 1$, and $v(\{2, 3\}) = v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. This is a simple game and Player 1 is the only veto player. How does the core of this coalition game looks like?

13. Prove: A player who is in every winning coalition in a simple game is called a veto player.A simple game has a non-empty core if and only it has veto players.

14. For those you did not attend my course in Praxis der Optimierung I have added a link to a vid that shows you how to use MS Execl Solver:

    https://www.youtube.com/watch?v=hjKwfjgK9_M

    (Some game theory examples are finally solved by an optimisation problem.)

15. Consider the Prisoner's Dilemma that we discussed in the lecture. However, suppose now that this game is repeated once and that both players

are informed of the outcome after the first round (just before the replay). Draw the game tree and consider how many subgames this game includes.

16. (Chomp Game). Start with an $m \times n$ array viewed as a chocolate bar, but with the lower left corner square poisened. On each turn, a player chooses a square and eats this square and all other squares that lie above and to the right of this square (i.e. northeast corner). The last player to eat a nonpoison square wins. Draw the game tree for a 3×2 chocolate bar.

17. Let $\mathbf{A}$ and $\mathbf{B}$ two $m \times n$ matrix games. If mixed strategy $\mathbf{p}$ guarantees the row player and in $\mathbf{A}$ an expected payoff greater equal r and in $\mathbf{B}$ an expected payoff greater equal s, prove that in the $\mathbf{A} + \mathbf{B}$ matrix game $\mathbf{p}$ guarantees the row player an expected payoff greater equal $r + s$.