18. Consider the following matrix game:

$$\begin{array}{c} \\ T \\ MU \\ MD \\ B \end{array} \begin{array}{c} \begin{array}{ccc} L & M & R \end{array} \\ \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 1 \end{pmatrix} \end{array}$$

- Determine the Maximin value and the Minimax value in **pure** strategies. What can you conclude from this about the value of the game?
- Does this game have a saddle point?
- Write down the LPs (Linear Programming Problems) for computing (mixed) Maximin and (mixed) Minimax strategies.
- If you want, you can also additionally solve the LPs by Excel Solver, GAMS, or Matlab etc.

19. Given is a $m \times n$ matrix game $\mathbf{A}$ with the (game) value v. Prove that, if you add a fixed constant c to all entries of $\mathbf{A}$, then the game game value of the new matrix game equals to $v + c$.

20. Suppose the spectrum of political positions is described by the closed interval (line segment) [0,5]. Voters are uniformly distributed over [0,5] (but assume that there are 10 (Million) voters to avoid fractions). There are two candidates, who may occupy any of the positions in the set $\{0, 1, 2, 3, 4, 5\}$. Voters will always vote for the nearest candidate. If the candidates occupy the same position they each get half of the votes. The candidates simultaneously and independently choose positions. Each candidate wants to maximize the number of votes for him/herself. Only pure strategies are considered.

- Model this situation as a bimatrix game between the two candidates.
- Determine pure best reply functions PB_i, and use this to
- determine all Nash equilibria (in pure strategies), if any.

21. Consider a $m \times n$ bi-matrix game $(\mathbf{A}, \mathbf{B}), \mathbf{p} \in \Delta^m$, and $\mathbf{q} \in \Delta^n$. Prove that

$$\mathbf{p} \in \beta_1(\mathbf{q}) \quad \Leftrightarrow \quad C(\mathbf{p}) \subseteq PB_1(\mathbf{q})$$
$$\mathbf{q} \in \beta_2(\mathbf{p}) \quad \Leftrightarrow \quad C(\mathbf{q}) \subseteq PB_2(\mathbf{p})$$

22. Determine and plot the best reply correspondences for the Battle of Sexes. Use the plot to determine all Nash equilibria.

23. Consider the following Bi-matrix game:

$$\begin{array}{c} \\ T \\ M \\ B \end{array} \begin{array}{c} \begin{matrix} W & Y & Z \end{matrix} \\ \begin{pmatrix} 2,2 & 2,2 & 0,0 \\ 1,0 & 2,4 & 1,5 \\ 0,4 & 3,0 & 3,3 \end{pmatrix} \end{array}$$

- Prove that M is strictly dominated by a combination of T and B
- Eliminate dominated M and continue to eliminate **strictly** dominated actions.
- Compute all Nash equilibria (uncountable many) Nash equilibria graphically.

24. Football European Championship 2020 Final: The Final between Austria and Germany is in the 90th minute. Each team has scored 3 goals as the referee whistles penalty for Austria. For Austria Alaba will compete, in the goal of the Germans is Neuer. Alaba has the two options to shoot in the left or in the right corner, Neuer has to choose one of the two corners. Of course, the decisions have to be made simultaneously. The two players know each other well from club training and they know exactly their chances (left - right seen from the center circle):

Alaba \ Neuer	left corner	right corner
left corner	0,7; 0,3	0,8; 0,2
right corner	0,9; 0,1	0,5; 0,5

If Alaba shoots for example into the left corner and Neuer also covers the left corner, the ball goes into the goal in 7 out of 10 cases, in the remaining 3 cases Neuer deflects the shot or the shot does not hit the goal.

Compute the best reply functions and plot them. Use the figure to determine Nash equilibria.