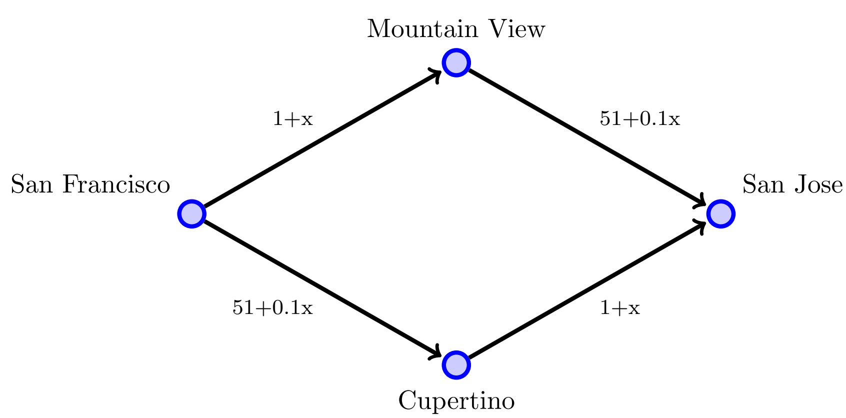
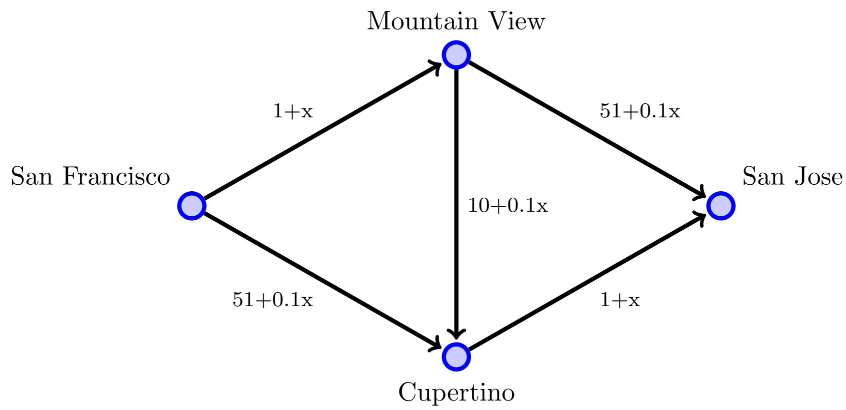
25. Braess Paradoxon. Consider a finite game with 60 players. There are two main roads connecting San Francisco and San Jose, a northern road via Mountain View and a southern road via Cupertino. Travel time on each of the roads depend on the number of x of cars using the road per minute, as indicated in the following diagram:

    For example, the travel time between San Francisco and Mountain View is 1 + x, where x is the number of cars per minute using the road connecting these cities, and the travel time between Mountain View and San Jose is 51 + 0.1x, where x is the number of cars per minute using the road connecting those two cities. Each driver chooses which road to take in going from San Francisco to San Jose, with the goal of reducing to a minimum the amount of travel time. Early in the morning, 60 cars per minute get on the road from San Francisco to San Jose (where we assume the travelers leave early in the morning so that they are the only ones on the road).



    What are all the Nash equilibria of this game? At these equilibria, how much time does the trip take at an early morning hour?

    The California Department of Transportation constructs a new road between Mountain View and Cupertino, with travel time between these cities 10 + 0.1 x. This road is one way, enabling travel solely from Mountain View to Cupertino.



Find a Nash equilibrium in this new game. Under this equilibrium how much time does it make to get to San Jose from San Francisco on an early morning hour? Does the construction of the additional road improve travel time?

Such a phenomenon was in fact noted in New York (where the closure of a road for construction work had the effect of decreasing travel time) and in Stuttgart (where the opening of a new road increased travel time).

26. A game $G = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ is called symmetric if

  (a) $S_i = S_j$, for each $i, j \in N$

  (b) the payoff functions satisfy

$$u_i(s_1, s_2, \ldots, s_n) = u_j(s_1, \ldots, s_{i-1}, s_j, s_{i+1}, \ldots, s_{j-1}, s_i, s_{j+1}, \ldots, s_n)$$

  for any vector of pure strategies $s = (s_1, s_2, \ldots, s_n)$ and for each pair of players satisfying $i < j$.

In a symmetric game, we call a strategy profile $\sigma = (\sigma_i)_{i\in N}$ that is a Nash equilibrium and satisfies $\sigma_i = \sigma_j \, \forall i, j \in N$ a symmetric (Nash) equilibrium.

The Volunteer's Dilemma: Ten people are arrested after comitting a crime. The police lack sufficent resources to investigate the crime thoroughly. The chief investigator therefore presents the suspects with the folowing proposal: if at least one of them confesses, every suspect who has confessed will serve a one-year jail sentence, and all the rest will be released. If no one confesses to the crime, the police will continue their investigation, at the end of which each one of them will recieve a ten-year jail sentence.

- Write down this situation as a strategic-form game, where the set of players is the set of people arrested, and the utility of each player (suspect) is 10 minus the number of years he spends in jail.

- Find all the Nash equilbria in pure strategies.
- Check if the game is symmetric. Try to find a symmetric equilibrium in mixed strategies.
- Suppose now the number of suspects is $n > 10$. Find a symmetric equilibrium in mixed strategies.