27. In the envelope game, there are two players and two envelopes. One of the envelopes is marked player 1, and the other is marked player 2. At the beginning of the game, each envelope contains one dollar. Player 1 is given the choice between stopping the game and continuing. If he chooses to stop, then each player receives the money in his own envelope and the game ends. If player 1 chooses to continue, then a dollar is removed from his envelope and two dollars are added to player 2's envelope. Then player 2 must choose between stopping the game and continuing. If he stops, then the game ends and each player keeps the money in his own envelope. If player 2 elects to continue, then a dollar is removed from his envelope and two dollars are added to player 1's envelope. Play continues like this, alternating between players, until either one of them decides to stop or k rounds of play elapsed (one round means that both players move). If neither player chooses to stop by the end of the k-th round, then both players obtain nothing. Assume players want to maximize the amount of money they earn.

    - Draw the game tree for k = 3.
    - Determine subgame perfect equilibrium by backward induction for k = 3.

28. Bidding Game. Players 1 and 2 bid for an object that has value 2 for each of them. They both have wealth 3 and are not allowed to bid higher than this amount. Each bid must be a nonnegative integer amount. Besides bidding, each player, when it is his turn, has the option to pass (P) or to match (M) the last bid, where the last bid is set at zero at the beginning of the game. If a player passes (P), then the game is over and the other player gets the object and pays the last bid. If a player matches (M), then the game is over and a player gets the object and pays the last bid with probability $\frac{1}{2}$. Player 1 starts, and the players alternate until the game is over. Each new bid must be higher than the last bid.

    - Draw the game tree of this extensive form game. How many subgames does this game have?
    - How many strategies does player 1 have? Player 2?
    - Reformulate as a normal form game and determine all pure Nash equilibria.
    - How many subgame perfect equilibria does this game have?
    - What is (are) the possible subgame perfect equilibrium outcome(s)?

    Remark: Note that if player 1 passes immediately at the beginning of the game the payoff is $(2, 5)$.

29. Ultimatum offer bargaining game. Two players negotiate over the price of a painting that player 1 sells to player 2. Player 1 proposes a price p to player 2. Then, after observing player 1's offer, player 2 decides whether to accept it (yes) or reject it (no). If player 2 accepts the proposal, then player 1 obtains p and player 2 obtains 100-p. If player 2 rejects the proposal, then each player gets zero.

    In this game, there are uncountable many trivial information sets for player 2. How does the set of actions look like in such an information set? Player 1 has uncountable many actions in her unique information set. However, the principle of Nash / subgame perfectness - discussed in finite games - can in a similar way applied to this infinite bargaining game.

    A cutoff-rule strategy for player 2 is a strategy where player 2 accepts any offer up to a $\underline{p}$, and rejects any offer higher than this threshold $\underline{p}$. Discuss, if a given offer $\hat{p} < 100$, let say $\hat{p} = 50$, combined with the cutoff-rule strategy with threshold $\underline{p} = \hat{p}$ defines a Nash equilibrium.

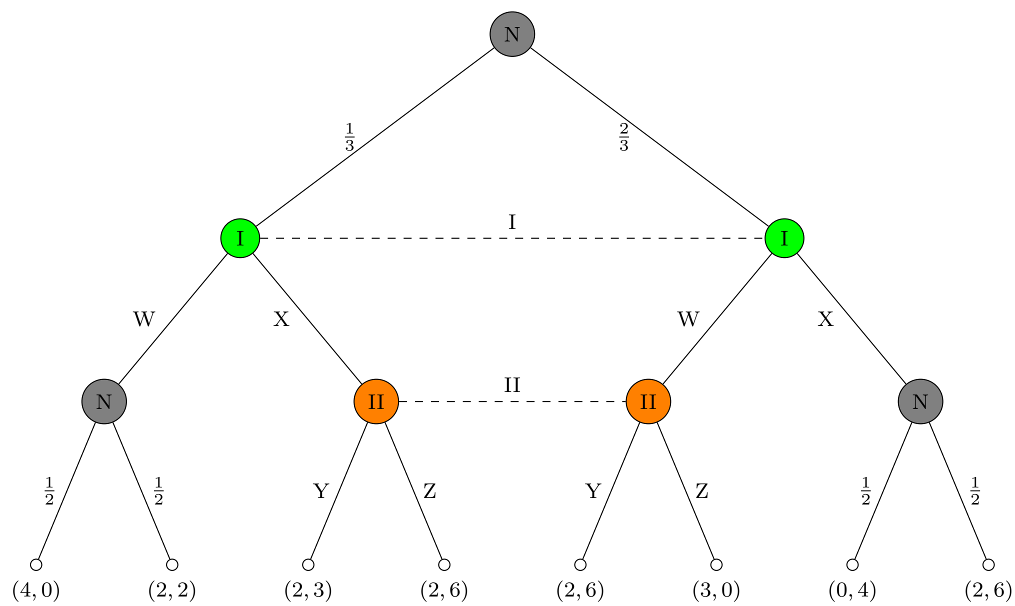
    Verify that there is a subgame perfect Nash equilibrium in which player 1 offers $p^* = 100$ and player 2 has the strategy of accepting any offer $p \leq 100$ and rejecting any offer $p > 100$.

30. Coin Game. At the start of play, Rose and Colin each put one chip in the pot as ante and each player tosses a coin. Rose sees the result of her toss, but not Colin's, and vice versa. It is then Rose's turn to play and she may either fold, ending the game and giving Colin the pot, or bet and place 2 more chips in the pot. If Rose bets, then it is Colin's turn to play and he may either fold, giving Rose the pot, or the may call and place 2 chips in the pot. In this latter case, both coin tosses are revealed. If both players have the same coin toss, the pot is split between them. Otherwise, the player who tossed heads wins the entire pot.

    - Draw the game tree of this extensive form game.
    - How many strategies does Rose have? Colin?
    - Reformulate as a normal form game and determine all pure Nash equilibria.
    - How many subgame perfect equilibria does this game have?

31. Consider the following game tree (next page):

    - How many subgames does this game have?
    - What are the information sets?
    - Reformulate this game in normal form
    - Derive all pure Nash equilibria. Which of them are subgame perfect?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