32. Continue with the game in Exercise 31.

    - (trick question; Fangfrage) Determine the Bayes consistent belief $(\alpha, 1-\alpha)$ for the information set of player 1.
    - If we denote the Bayes consistent belief for the information set of player 2 by $(\beta, 1-\beta)$, what is a rational decision for player 2?
    - Assume a mixed strategy $(\frac{4}{5}, \frac{1}{5})$ for player 1. Compute a Bayes consistent belief value β for the information set of player 2.
    - We have already verified that WY is subgame perfect. Is this strategy profile a perfect Bayesian equilibrium?

33. In this game in a strategic setting, players 1 and 2 play according to the matrix shown. However, player 1's payoff number x is private information. Player 2 knows only that x = 12 with probability $\frac{2}{3}$ and $x = 0$ with probability $\frac{1}{3}$.

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} x, 9 & 3, 6 \\ 6, 0 & 6, 9 \end{pmatrix} \end{array}$$

    Note that the matrix pictured is not the true normal form of the game, because player 1 observes x before making his decision. Player 1 observes nature's action before selecting between U and D, yet player 2 must make her choice without observing player 1's type or action.

    - Draw the game tree
    - Determine the information sets
    - Compute for all 4 pure strategies of player 1 beliefs for player 2
    - and picture the normal form of this game
    - Determine the pure Nash equilibrium
    - Is this Nash equilibrium perfect Bayesian?

34. The milk-whisky game. Consider the following signaling game. In a saloon, a stranger (player 1) is sitting at the bar; the stranger is either "weak" or "strong". In the saloon, the local hero (player 2) annouces that in the past strangers were "weak" with probability $\frac{1}{10}$. Player 1 knows if he is weak or not – player 2 does not; but due to the announcement the probabilities of either type of player 1 are common knowledge among the two players. Player 1 has two actions: either have milk (M) or have whisky (W) for breakfast. Player 2 observes the breakfast of player 1 and then decides to duel (D) or not to duel (N) with player 1. The payoffs

are as follows. If player 1 is weak and drinks milk then D and N give him payoffs of 1 and 3, respectively, if he is weak and drinks whisky, his payoffs are 0 and 2, respectively. If player 1 is strong, then the payoffs are 0 and 2 from D and N, respectively, if he drinks milk; and 1 and 3 from D and N, respectively, if he drinks whisky. Player 2 has payoff 0 from not dueling, payoff 1 from dueling with the weak player 1, and payoff -1 from dueling with the strong player 1.

- Draw the game tree
- Compute all pure strategy Nash equilibria of the game
- Find out which of these Nash equilibria are perfect Bayesian equilbria
- Give the corresponding beliefs and determine whether these equilibria are pooling or separating.