38. Consider a symmetric game given by following payoff matrix (version of stag hunt):

$$A = \begin{array}{c} \\ Stag \\ Hare \end{array} \begin{array}{c} \begin{array}{cc} Stag & Hare \end{array} \\ \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \end{array}$$

Write down the replicator dynamics and analyse the steady states.

39. In the early days of personal computing, people faced a dilemma. You could buy a computer running Microsoft Windows, or running Apple Mac OS. Either was reasonable satisfactory, although Apple's was better. However, neither type of computer dealt well with files produced by the other. Thus if your coworker used Windows and you used Apple, not much got accomplished.

We model this situation as a symmetric two-player game in normal form. The strategies are buy Microsoft or buy Apple. The payoffs are given by the following matrix (version of coordination game).

$$A = \begin{array}{c} \\ Microsoft \\ Apple \end{array} \begin{array}{c} \begin{array}{cc} Microsoft & Apple \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$$

- Please write down the replicator dynamics for buy Microsoft.
- Proof that $\frac{2}{3}$ is a steady state.

The mixed strategy profile $((\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3}))$ is a symmetric Nash equilibrium. One feels it does not correspond to any behavior one would ever observe. Even if for some reason people picked computers randomly, why would the choose the worse computer with higher probability?

To resolve this mystery, we imagine a larger population of people who randomly encounter each other and play this two-player evolutionary game. People observe which strategy, buy Microsoft or buy Apple, is on average producing higher payoffs. They will tend to use the strategy that they observe produces the higher payoff.

Looking at the phase protrait of the replicator dynamics, we see that $\frac{2}{3}$ is unstable. It is a kind of threshold, deviding the field into two basin of attraction for the two evolutionarily stable strategies buy Microsoft and buy Apple. The result is, if the share of Microsoft user is higher than $\frac{2}{3}$ you should buy Microsoft, otherwise buy Apple.

40. In the following game, you are the (benevolent) owner of 3 CNC machines M_i. Three customers want to execute their jobs $J_i, i = 1, 2, 3$ where each job can be processed on any machine, but each machine processes only one job. However, the machine M_j is originally assigned to the $j-$th player or his job M_j, respectively; only within coalitions players are allowed to swap machines.

    The parameter $k_i j$ denotes the cost player i has to pay when job J_i is performed on the machine M_j. For the cost of a coalition then arises

    $$c(S) = \min_{\alpha \in \alpha(S)} \sum_{i \in S} k_{i\,\alpha(i)}$$

    where $\alpha(S)$ is the set of all permutations of the machines of the coalition S (machines not from S are not permuted), and $\alpha(i)$ is the $i-$th element of the permutation α.

    For the following combined cost saving and permutation game with 3 jobs and 3 machines, compute the values (worth) of all possible coalitions:

k_{ij}	CNC 1	CNC 2	CNC 3
Job 1	1	2	4
Job 2	3	6	9
Job 3	4	8	12

41. Consider a three person coalition game (N, v) given by the following characateristic function:

    $$v(S) = \begin{cases} 0 & S = \emptyset \\ 1 & S = \{1\} \text{ or } \{2\} \\ 2 & S = \{3\} \\ 4 & |S| = 2 \\ 5 & |S| = 3 \end{cases}$$

    Is this coalition game superadditiv?

    Determine the imputation set for this coalition game.

42. For every real number a the 3-person coalition game v_a is given by $v_a\{i\} = 0, i = 1, 2, 3, v_a\{1, 2\} = 3, v_a\{1, 3\} = 2, v_a\{2, 3\} = 1$, and $v_a\{1, 2, 3\} = a$.

    - Determine the minimal value of a so that the TU-game v_a has a nonempty core.
    - Calculate the Shapley value of v_a for $a = 6$.
    - Determine the minimal value of a so that the Shapley value of v_a is a core distribution (allocation).

43. Suppose that two players (bargainers) bargain over the devision of one unit of a perfect divisible good. We denote the utility function of player 1 by $u_1(\alpha)$, $0 \leq \alpha \leq 1$ with u_1 increasing in α and $u_1(0) = 0$ and $u_1(1) = 10$ (note that u_1 is not necessarily linear). The utility function of player 2 $u_2(\alpha) = 2\,u_1(\alpha)$. Formulate the Nash bargaining problem and solve it.