1. Übung Mathematische Statistik WS15

1. Find a sufficient statistic for the parameter p of an alternative distribution.
2. Find a pair of sufficient statistics for the gamma distribution with density
$$\frac{\lambda^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x}[x \geq 0].$$
3. Calculate the ML estimator for a Poisson distribution and show that it is efficient.
4. Calculate the ML estimator for the mean of a normal with known variance distribution and show that it is efficient.
5. Calculate the Kullback-Leibler Information between two normal distributions.
6. Calculate the Kullback-Leibler Information between two exponential distributions.
7. $(X_1, \ldots, X_n)$ is a sample of a uniform distribution on $[0, \theta]$.
   (a) Calculate the ML estimator and its mean and variance. How can it be modified so that it becomes unbiased?
   (b) Calculate the moment estimator, its mean and variance. How does this compare to the modified ML estimator?
   (c) Show that the modified ML estimator is the only unbiased estimator that is a function of the sufficient statistic and hence efficient.
8. Construct a confidence interval for the Parameter θ in the previous example.
9. $(X_1, \ldots, X_n)$ is a sample from a distribution with density
$$\frac{2x}{\theta^2} e^{-x^2/theta^2}[x \geq 0].$$
   Calculate the ML and moment estimators, modify them if necessary so that they become unbiased, and compare their variances.
10. $(X_1, \ldots, X_n)$ is a sample from a uniform distribution on $[\theta, 2\theta]$.
    (a) Calculate the ML estimator and modify it so that it becomes unbiased.
    (b) Show that
$$\frac{1}{3}(\min(X_1, \ldots, X_n) + \max(X_1, \ldots, X_n))$$
    is another unbiased estimator.
    (c) Compare the variances of these estimators.